NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML STANDARDS BRIEF

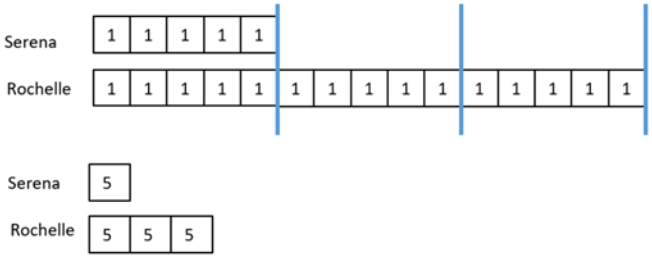
Multiplicative Comparisons- Grade 4

MULTIPLICATIVE COMPARISONS: AN OVERVIEW

In Grade 4 students extend their work from third grade related to interpreting, representing, and solving multiplication and division situations involving equal groups. Grade 4 focuses heavily on exploring word problems about multiplicative comparisons. A multiplicative comparison is a situation in which there are two quantities related by a factor. Multiplicative comparisons always involve two quantities and a factor that relates the two. This is different from additive comparison situations which involve two quantitites and the referent about how much more or less one quantity is compared to the other. These situations can involve discrete sets of objects (e.g., basketballs, people, pencils), units of measurement (lengths, volumes, masses), or other quantities.

REPRESENTATIONS OF MULTIPLICATIVE COMPARISON SITUATIONS

Multiplicative comparison situations can be represented in multiple ways. Consider the task, *Serena has 5 jelly beans. If Rochelle has 3 times as many jelly beans how many does Rochelle have?*



The representations above are visual models sometimes refered to as bar diagram (Charles, 2012; Diezmann & English, 2001). In the first representation each jelly bean is represented with a box in the bar diagram. The bar diagram supports students determination of the product using counting strategies. Drawing on (something from 3rd grade?)…. students should be able to generate the first representation and through discussion reason that Rochelle's number of jelly beans can be represented as 5 + 5 + 5 = 15 or 3 x 5 = 15. The second representation shows a more sophisticated model as Serena's jelly beans are represented with a bar of 5, and Rochelle has 3 copies of Serena's quantity, which in this case is 3 groups of 5.

QUESTIONS TO CONSIDER WITH COLLEAGUES

How can you extend student's understanding of multiplication and division to support their work with multipicative comparisons?

What mathematical representations make sense to support work with multiplicative comparisons?

As students embark on exploring multiplicative comparison situations, it is important to consider research on how students make sense of multiplicative problem solving situations. Research from the Cognitively Guided Instruction (CGI) project (e.g., Carpenter et al., 2014) identified these strategies: **direct modeling, counting strategies,** and **derived facts**.

Direct modeling would include the use of concrete materials (counters, cubes) or representations, like bar diagrams, to model the situation. These are sometimes refered to as concrete or representational approaches. Counting strategies would include skip counting (e.g., Serena: 5; Rochelle: 5, 10, 15). Approaches utilizing derived facts focus on the idea that 3 times more than 5 can be represented as 3 x 5 = 15. Ideally, teachers pose tasks and provide opportunities for students to choose which strategies they use, while looking for ways to eventually move students away from direct modeling when they are ready.

THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION

When exploring multiplicative comparison situations there are rich opportunities to have students work with, reason about, and discuss the relationship between multiplication and division (4.OA.1,, 4.NBT.6). For multiplicative comparison situations in which the first quantity (5 hair bows) and the referent (3 times as much) indicates that the unknown quantity being larger than the first quantity, these situations intuitively are solved using multiplication strategies or repeated addition (5+5+5=15 or 5x3=15).

Meanwhile, in situations in which the referent indicates that the unknown quantity is smaller than the first quantity, these situations could be solved using either multiplication or division. Consider*, Allison has 12 crayons. If Allison has 3 times more crayons than Toby how many crayons does Toby have?* Direct modeling approaches for the first equation could include grouping 12 manipulatives, such as cubes, 1 at a time into 3 groups. When all 12 objects have been divided evenly into 3 groups, the number of objects in each group, 4, represents Michelle's number of bracelets.

A method using counting strategies could include skip counting by 3s up to 12 and reasoning that since 12 is the 4^{th} multiple then the missing number is 4. Students using derived facts strategies would draw on the knowledge that 4 x 3 = 12 or 12 ÷ 3 = 4.

MULTIPLICATIVE COMPARISONS WITH FRACTIONS

Grade 4 students multiply a fraction or mixed number by a whole number (NC.4.NF.4). To that extent it is appropriate for students to solve tasks in which they have a fraction quantity and a whole number representing how many times larger a different quantity is. For example, *Alonzo has 1 and ½ Liters of water in his container. If Tim has three times as much water how many Liters of water does Tim have?* which can be written as 1 ½ x 3 = ___. Likewise Grade 4 students should also explore situations in which there is a whole number quantity and the comparison quantity is a fraction. *For example, Bralyn has 12 pencils. If Angela has 2/3 as many pencils as Brayln how many pencils does Angela have?*, which can be written as 12 x 2/3 = __.

When posing tasks that involve multiplicative comparison situations with fractions, though, remember that the only operation that should be done involve a whole number multiplied by a fraction or mixed number. For example, fourth graders should not explore situations like *Mary has 4 ½ gallons of juice. If Mary has two times as much juice as Briana, how much juice does Briana have?* This situation is not appropriate for Grade 4 students since students would have to divide 4 ½ by 2 or multiply 4 ½ by ½. Teachers are encouraged to be careful when selecting, modifying, or creating tasks involving multiplicative comparison situations (Empson & Levi, 2011).

NUMBER SIZE

In Grade 4 the Standards call for students to multiply up to two-digit by two-digit numbers and up to four-digit numbers by a one-digit number (NC.4.NBT.5). Further, Grade 4 students are expected to divide a four-digit number by a one-digit number (NC.4.NBT.6). While students are expected to solve multiplicative comparison problems involving these large numbers by the end of the year, the rigor in multiplicative comparison situations is intepreting the situation and determining how to solve it. Therefore, the initial exploration of these problems should focus on smaller numbers to support direct modeling and counting strategies (Carpenter et al., 2014; Empson & Levi, 2011). As students become more familiar with multiplicative comparison situations, students will be able to use more derived facts strategies with larger numbers.

Early exploration of multiplicative comparison situations should focus on having the student interpret the comparison, determine an appropriate way to represent the comparison, solve the problem, and reason about whether or not their answer makes sense. This work lays the groundwork for ratios and proportions in middle grades.

REFERENCES

Carpenter, T., Fennema, E., Franke, M., Levi, L. and Empson, S. B. (2014).. *Children's Mathematics, Second Edition: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

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