Next Steps and Instructional Moves

The intended purpose of this document is to provide teachers with a tool to determine student understanding and suggest instructional moves that may help guide a student forward in their learning of a particular concept or standard. This guide is not an exhaustive list of strategies.

Numbers and Operations – Fractions
Extend Understanding Operations of Fractions and Decimals

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

NC.4.NF.3 Understand and justify decompositions of fractions with denominators of 2, 3, 4, 5, 6, 8,10, and 12, and 100.
- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.

Use unit fractions to understand operations of fractions.

NC.4.NF.4 Apply and extend previous understanding of multiplication to:
- Model and explain how fractions can be represented by multiplying a whole number by a unit fraction, using this understanding to multiply a whole number by any fraction less than one.
- Solve word problems involving multiplication of a fraction by a whole number.

Understand decimal notation for fractions, and compare decimal fractions.

NC.4.NF.6 Use decimal notation to represent fractions.
- Express, model and explain the equivalence between fractions with denominators of 10 and 100.
- Use equivalent fractions to add two fractions with denominators of 10 or 100.
- Represent tenths and hundredths with models, making connections between fractions and decimals.

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	Students that are consistently scoring "Not Yet" on fraction tasks could have a variety of errors. Students consistently scoring "Not Yet" do not have any effective strategy for decomposing fractions. They also may have difficulty identifying and understanding unit fractions. This could be due to students lack of understanding of benchmark fractions.
Not Yet	<u>Next Steps:</u> **For students having trouble identifying unit fractions:** ● Move students back to manipulatives (using fraction bars, pattern blocks, fraction tiles, or fraction strips) to help students visualize the value of fraction. Have student work with the manipulatives to physically separate the different parts into different groups. **For students having trouble decomposing a fraction:** ● Begin with manipulatives that start with unit fractions that are combined, such as $$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$
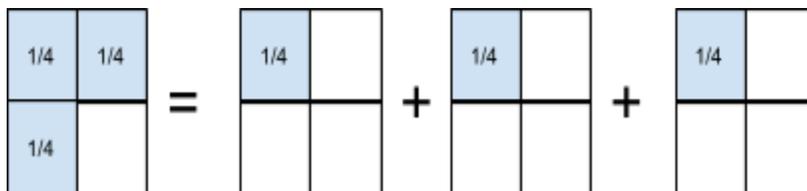
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- Once students become familiar with decomposing fractions into unit fractions, move towards more non-traditional models to help students understand that fractions can be decomposed in in multiple ways, such as
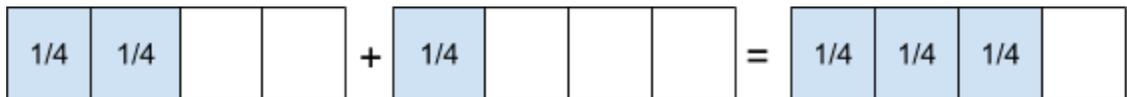
$$\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$$

1/4	1/4
1/4	

$=$

1/4	

$+$

1/4	
1/4	
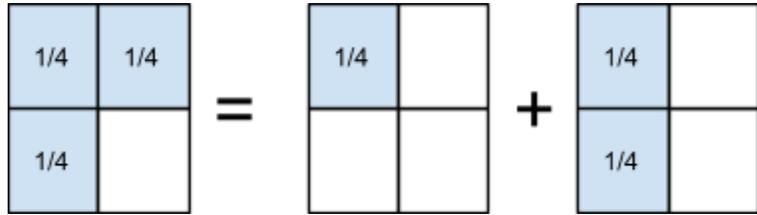
For students having trouble understanding how to add and subtract fractions:

- Much like decomposing fractions, students will work with manipulatives to add and subtract fractions, such as

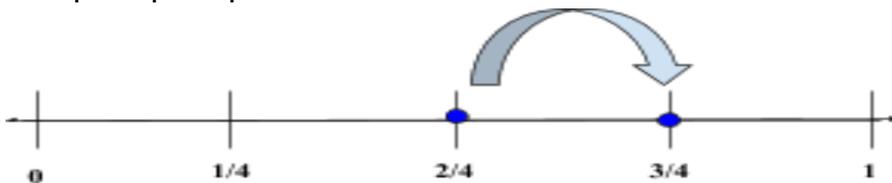$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

| 1/4 | 1/4 | | |

$+$

| 1/4 | | | |

$=$

| 1/4 | 1/4 | 1/4 | |

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$
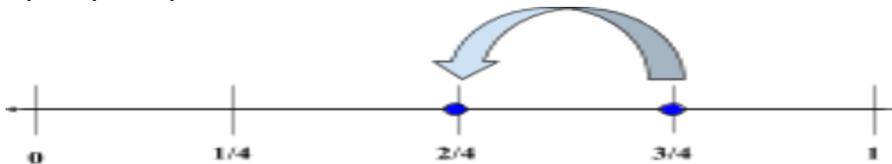
| 1/4 | 1/4 | 1/4 (X) | |

$=$

| 1/4 | 1/4 | | |

- Use a number line to add or subtract the fractions. Using a number line may provide another visual that can help students. For example,

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

0   1/4   2/4   3/4   1

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$


0   1/4   2/4   3/4   1

- Have students go back to focusing on finding an estimate before actually solving an addition or subtraction problem. Students should look at the fractions and reason about their size. Are their fractions more than 1/2 , more than 1, or less than ½? Using these benchmark fractions should help students to realize that if both of the fractions are more than ½ than their answer

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should be more than one whole. To help students estimate, ask: "How might you think about this without actually calculating an answer?"

For students having trouble comparing fractions with tenths and hundredths as denominators:
- Use decimal grids and place value blocks to visually represent the relationship between tenths and hundredths as it pertains to fractions and decimals.
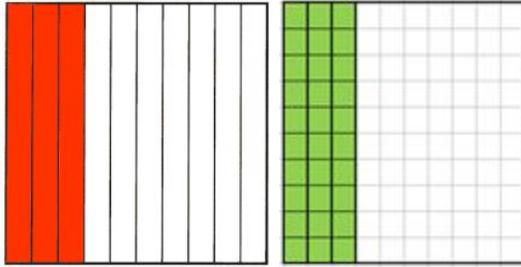
$$\frac{3}{10} = \frac{30}{100}$$

$$\frac{3}{10} = 0.3 \qquad \frac{30}{100} = 0.30$$

$$0.3 = 0.30$$

- As you begin to make connections between fractions and decimals make sure students are correctly naming the decimal to reinforce the place value of each digit. For example, make sure students do not get into the habit of saying "five point four" rather than "five and four tenths" when reading a decimal.
- Use place value charts to have students model writing numbers to determine value.

Hundreds	Tens	Ones	●	Tenths	Hundredths
			●	3	
			●	3	0

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	Students that are consistently scoring "Progressing" have a strategy for adding and subtracting fractions; however, they have not mastered this strategy or may also still struggle with the conceptual idea of unit fractions and decomposition of fractions. Students also may not understand how to decompose fractions to make similar addition and subtraction problems. Students may also be able to solve problem with tenths and hundredths, but they may not conceptually understand the relationship between tenths and hundredths.

Progressing	<u>**Next Steps:**</u>
	For students who add or subtract the denominator when they are adding or subtracting fractions:
	● Use pictures and models to show that you still have the same size pieces or the same number of pieces in each whole, but all that is changing is the amount of pieces selected. Ask:
	   ○ Can you create a model for each of these fractions?
	   ○ How can a model help you determine whether the sum will be more or less than 1?
	● Have students go back to focusing on finding an estimate before actually solving. Students should look at their fractions and reason about their size. Are the fractions more than 1/2 , more than 1, or less than ½? Using these benchmark fractions should help students to realize that if both of the fractions are more than ½ than their answer should be more than one whole. To help students estimate, ask: "How might you think about this without calculating an answer?"
	● Make sure when you are prompting students to explain their answer that you ask students "What fraction will you have now?" as opposed to asking "How many pieces do you have now?" - You want students to answer with a fraction and not to be further confused about the number of pieces.
	
	For students who struggle when the sum is greater than 1:
	● Begin with a visualization to model how students are creating a whole. Often times having students shade while they add will help them see that a whole has been created. For example,
	
	$$\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$
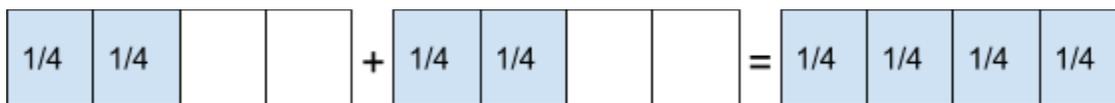	
	● Students may feel more comfortable using a number line to show this.
	
	$$\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$
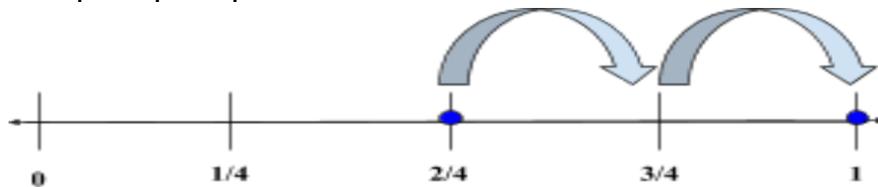	
	● Students can build on this by using a model or manipulatives to add fractions that will give them a sum greater than one. Student should see with their model or with their manipulatives that the sizes of the pieces are not changing and that they have one entire whole. This can help to build students' understanding of improper fractions and mixed numbers.

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$$\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$$
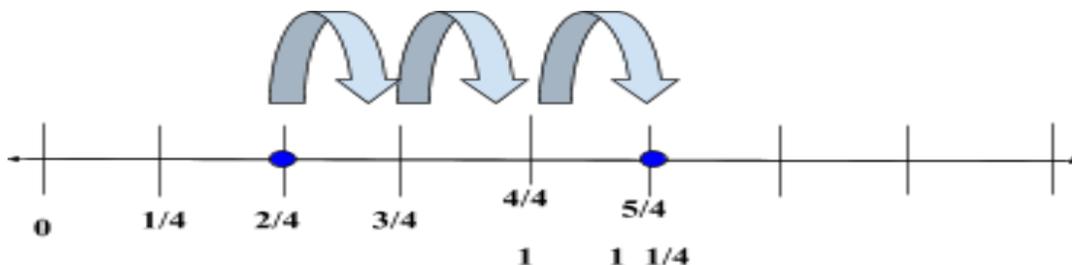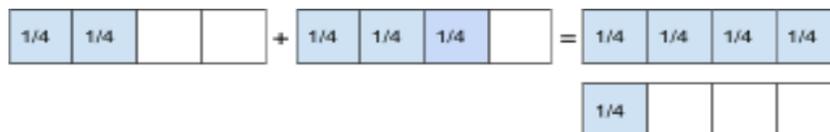
For students who struggle when subtracting mixed numbers:
● Have students draw a visual representation of the subtraction problem. Have students eliminate what is being taken away so that can see what remains.

$$2\frac{1}{4} - \frac{3}{4} = 1\frac{2}{4}$$
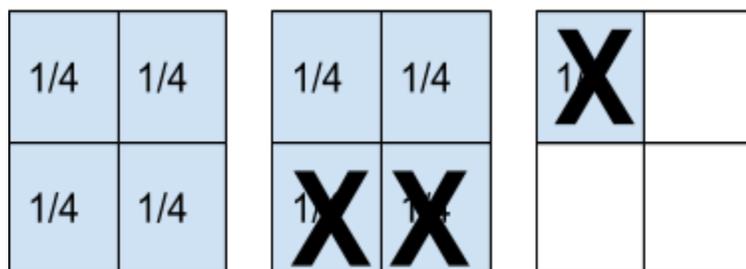
● Have students connect their visual to decomposing fractions and their knowledge of a whole to be able to subtract fractions using numbers.

$$2\frac{1}{4} - \frac{3}{4} = ?$$

● Students need to think about decomposing two wholes. This could instead be written as $2 = 1 + \frac{4}{4}$. If students add $\frac{4}{4} + \frac{1}{4}$ they will get $\frac{5}{4}$ which they can then subtract $\frac{3}{4}$. Students then have the problem $1\frac{5}{4} - \frac{3}{4} = 1\frac{2}{4}$.

For students who multiply the whole number by both the numerator and the denominator of the fraction:
● Have students explain what 1 whole looks like as a fraction versus a whole number with a denominator of one. Go back to illustrations of these fractions or pull out fraction tiles. We want students to visualize the difference between 3/3 versus 3/1.
● Ask students to go back to the connection of repeated addition and multiplication. Have students think about how with whole number multiplication 2 x 3 = 6. This is 2 groups of 3 which equals 6. Then have students apply this thinking to their fraction. If they have 3 x ⅔ then they need to realize that they are repeatedly adding ⅔ three times or that they should have three groups of two-thirds. Have students start with this connection and repeated addition and then students should be able to connect this to multiplication.

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$3 \times \frac{2}{3} = 2$
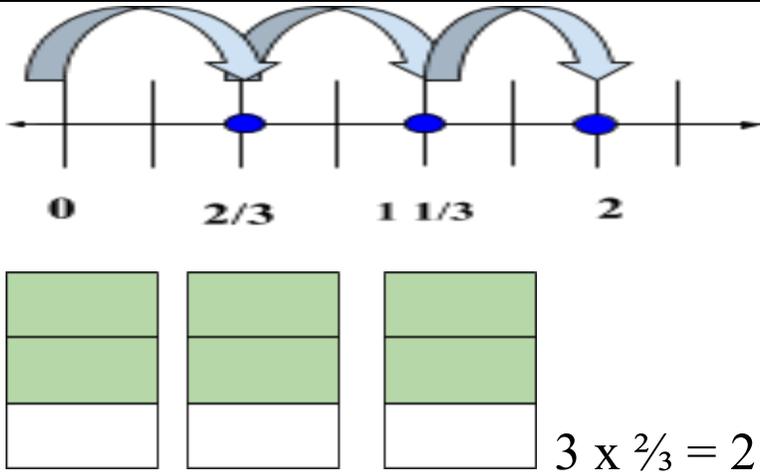
- Make sure students are not struggling with the multiplication aspect by skip counting or listing the multiples when multiplying fractions. Have students record using models and equations to represent the repeated addition.

For students that are still struggling with making the connection between tenths and hundredths:
- Review concepts about equivalent fractions from previous clusters.
- Review base ten knowledge and have students relate their knowledge of the relationship between tens and hundreds to tenths and hundredths to multiply a fraction by a whole number equal to 1. For example,

$$\frac{1}{10} \times \frac{10}{10} = \frac{10}{100}$$

Meets Expectation	Students that are consistently scoring "Meets Expectation" on decomposition of fraction tasks should have a good conceptual understanding of the true meaning of a unit fraction and decomposing of fraction. Students may need to work on developing fluency with multiple ways to decompose a fraction and the conceptual relationship between tenths and hundredths.
	Next Steps: • Ask students to explain how they can determine all the different ways to decompose a fraction based on unit fractions. • Ask students to decompose fractions other than just breaking apart into unit fractions. • Have students create anchor charts to express how to add and subtract mixed numbers using various strategies. They will need to also be able to explain these strategies to others in the class.

The following books were referenced for ideas about teaching misconceptions and/or would be great resources for teachers looking for more next step ideas:

Math Misconceptions - PreK - Grade 5 by Bamberger, Oberdorf, and Shultz-Ferrell

Activities to Undo Math Misconceptions Grades 3-5 by Bamberger, Shultz-Ferrell

Uncovering Student Thinking in Mathematics - Grades K-5 by Tobey, Minton

Mine the Gap for Mathematical Understanding - Grades 3-5 by SanGiovanni

Uncovering Student Thinking About Mathematics in the Common Core - Grades 3-5 by Tobey, Fagan