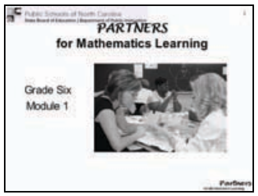
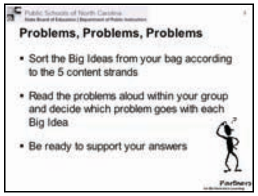
General Materials and Supplies:

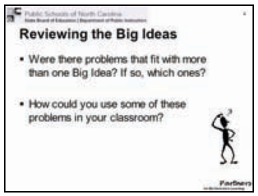
Laptop, Projector, Power Cord	String	*What's in a circle?* Handout
Sets of Problems, Problems, Problems Cards	Rulers	cm Grid Paper
Sets of Big Ideas Cards	Measuring Tapes	Big Ideas Handout
Fiddling with Factors Handout	Chart Grid Paper	A Variety of Circular Objects (with diameter less than 15 cm)
Calculator	Markers	Square tiles
Patty Paper	Small Sticker Dots	Essential Standards Handout

Slide	Tasks/Activity	Personal Notes
	(slide 1) **Module One** This module focuses on Problem-Solving with an emphasis on Number, Geometry, and Data Analysis. ▪ Introductions ▪ Establish ground rules	
	(slide 2) **Video Overview** Welcome to Partners for Mathematics Learning Professional development. This module, like others that will follow, relates the big ideas of K-8 mathematics to North Carolina's new 2009 Essential Standards for your grade level. Our goal is to assist you in helping your students develop a deep understanding of the mathematics they need to learn. These modules include strategies for developing problem solving and mathematical reasoning, discussions of important content, and suggestions for assessing student thinking. As we work together to increase student achievement in mathematics, we recognize the numerous challenges that teachers face. It is our hope that within this professional development you will find support for helping all students become fluent with mathematical concepts and procedures. Thank you for being part of North Carolina's continued progress toward mathematical excellence.	



(slide 3) **Problems, Problems, Problems**
Participants will work in table groups to complete this activity.

- Give each group a set of problem cards and a set of Big Idea cards (Module One, Handouts One and Two)
- Ask groups to sort the Big Idea cards according to the 5 content strands (Number and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebra)
- Ask the groups to choose a reader. The reader will read the problem and the group will decide which Big Idea matches the problem.



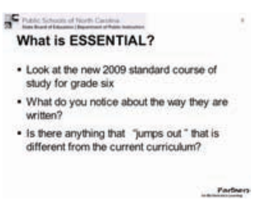
(slide 4) **Reviewing the Big Ideas**
A whole group discussion should follow focusing on the following questions:

Were there problems that fit with more than one Big Idea? If so, which ones?

- These answers will vary
- Be careful not to discredit any group's matches
- Encourage participants to justify their findings

How could you use some of these problems in your classroom?

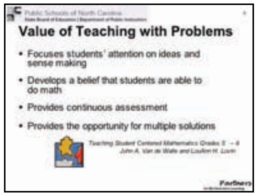
- Warm-up problems
- Assessments
- Have students write multiple choice answers for these problems
- Think, Pair, Share
- Journal entries



(slide 5) **What is Essential?**
Introducing the new curriculum…

- *Look at the new standards for grade 6*
- *What do you notice about the way they are written?*
- *Is there anything that "jumps out" as very different from the current curriculum?*

Note to participants that this is a first-glance look, there will be time later for a closer look, so just consider the structure and noticeable changes.



(slide 6) **Value of Teaching with Problems**

Ask for a volunteer to read the points from Van de Wall.

"Problems" refers to tasks that engage students and lead to conceptual understanding.

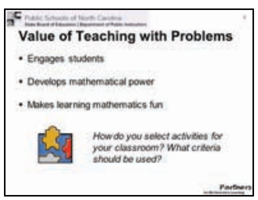
Research from ***Teaching Student Centered Mathematics Grades 5 – 8***
John A. Van de Walle and LouAnn H. Lovin

Focus students' attention on ideas and sense making
- Fewer directions leads to more math

Develops a belief that students are able to do math
- More problems students solve, the more confidence they have in their mathematical abilities

Provides continuous assessment
- As students discuss ideas, they provide a wealth of information regarding understanding

Provides the opportunity for multiple solutions
- Students bring different experiences to a problem solving tasks
- Students make different connections and solve problems in a variety of ways



(slide 7) **Value of Teaching with Problems**

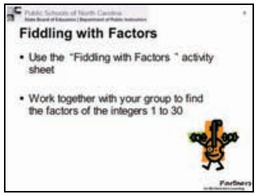
Engages students
- "Real learning is engaging, whereas following directions is often boring."

Develops mathematical power
- The processes of doing math are just as important as the doing of mathematics. (NCTM Process Standards)

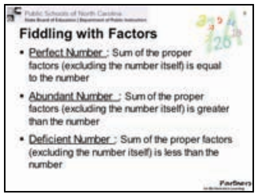
Makes learning math fun
- Teaching by doing is often more fun than teaching by telling.
- "The excitement of students developing understanding through their own reasoning is worth all the effort."

How do teachers select activities for the classroom?
What criteria should be used?

Ask participants this question; provide ample time for brainstorming. On chart paper, create a list of criteria for choosing activities.



(slide 8) **Fiddling with Factors**
Ask participants to get out the Fiddling with Factors handout (Module One, Handout Three) and ask them to work together in their groups to find the factors of the integers from 1 to 30. Monitor to see that groups understand the concept of factor and are listing all factors. Pay attention to strategies that are being used to list factors (factor trees, multiplication pairs, lists, building arrays with square tiles, etc.).



(slide 9) **Fiddling with Factors**
Perfect number – *sum of the factors excluding the number itself is equal to the number*

- The first perfect number is 6, 1 + 2 + 3 = 6
- The second perfect numbers is 28, 1 + 2 + 4 + 7 + 14 = 28
- The next three perfect numbers are 496, 8128, and 33,550,336
- The first four perfect numbers were the only ones known to early Greek mathematicians

Abundant number – *sum of the factors excluding the number itself is greater than the number*

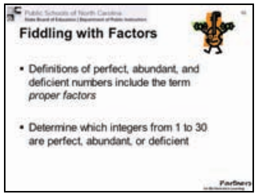
- Also known as an excessive number
- The abundant numbers between 0 and 100 are 12, 18, 20, 24, 30, 36, 40, 42, 48, 54, 56, 60, 66, 70, 72, 78, 80, 84, 88, 90, 96, and 100
- The smallest abundant number not divisible by 2 is 945

Deficient number – *sum of the factors excluding the number itself is less than the number*

- Also known as a defective number
- The first 10 deficient numbers are 1, 2, 3, 4, 5, 7, 8, 9, 10, and 11.

Note to participants that these definitions apply only to integers. These terms are often used in *Math Counts* and other math competition materials.

It is important to provide students with number activities that spark curiosity. Giving students the opportunity to investigate types of numbers provides them with math investigations that may extend beyond the classroom.



(slide 10) **Fiddling with Factors**
Definitions of perfect, abundant, and deficient numbers include the term *proper factors*.
However, there are many different definitions of the term.

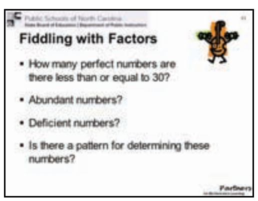
Definition #1: The proper factors of 6 are 1, 2, and 3

Definition #2: The proper factors of 6 are 2 and 3

Definition #3: The proper factors of 6 are 1, 2, 3, and 6.

To determine perfect, abundant, and deficient, mathematicians agree that all factors except the number itself are used to determine the sum.

Now determine which integers from 1 to 30 are perfect, abundant, and deficient.
Complete Part 2 of the *Fiddling with Factors* handout.
1. 6, 12, and 5
2. 28, 25, and 12
3. 12 and 11
4. 30 and 6
5. 15 and 20
Discuss with your group, how you determined your answers for each question.



(slide 11) **Fiddling Factors**
Ask participants the following questions:
How many perfect numbers are there less than or equal to 30? **2**

How many abundant numbers are there less than or equal to 30? **5**

How many deficient numbers are there less than or equal to 30? **23**

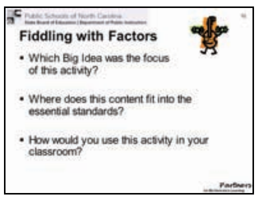
Is there a pattern for determining these numbers?
Note that these three categories are disjoint. So that for a set of 50 integers, the total from all categories should equal 50.

Note: The following information is for teacher knowledge only, not intended for students, nor is *it intended for teachers to discover.*

Euclid discovered that the first four perfect numbers are generated by the formula $2^{n-1}(2^n - 1)$:

for $n = 2$: $2^1(2^2 - 1) = 6$
for $n = 3$: $2^2(2^3 - 1) = 28$
for $n = 5$: $2^4(2^5 - 1) = 496$
for $n = 7$: $2^6(2^7 - 1) = 8128$.

There are no apparent patterns for determining the next deficient or abundant numbers. Students and participants may try to find patterns. Students should be able to recognize when there is no pattern as well as when there is a pattern. Any pattern suggested might be challenged by a counterexample. This could be a fun competition for students (pattern busting).



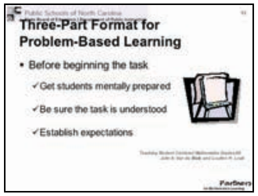
(slide 12) **Fiddling with Factors**
Which Big Idea was the focus of this activity?

Expanded understanding and use of classes of numbers increases students' abilities to describe situations and solve problems

Developing an understanding of factors is extremely important for working with operations with fractions in the middle grades and working with polynomials in Algebra I.

How would you use this activity in your classroom?
- Modify assignment using less numbers
- Assessment: listening to students talk in their groups gives teachers valuable information about factor knowledge
- Meaningful Practice
- Enrichment

Where does this content fit in the Essential Standards?
Answers may vary, but allow participants time to look at the Standards for 6th grade.

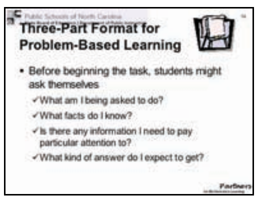


(slide 13) **Three-Part Format for Problem-Based Learning**
Have a volunteer read the bullets on the following slides.

Research from ***Teaching Student Centered Mathematics Grades 5 – 8***
John A. Van de Walle and LouAnn H. Lovin

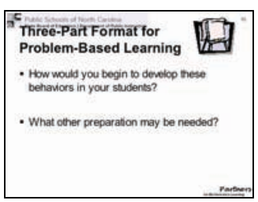
<u>Before beginning the task</u>

- *Get students mentally prepared* – begin with a simple version of the task, brainstorm solutions, use mental math to estimate answers
- *Be sure the task is understood* – help the students clarify what the problem is asking, have students restate the problem in their own words, review vocabulary
- *Establish expectations* – "Every task should require more of students than simply the answer." – make clear to students when they will be required to explain, write, or discuss



(slide 14) **Three-Part Format for Problem-Based Learning**
Before beginning the task, students might ask themselves

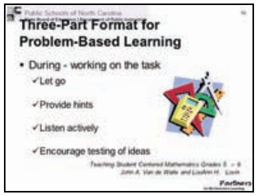
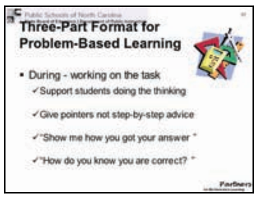
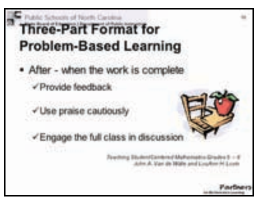
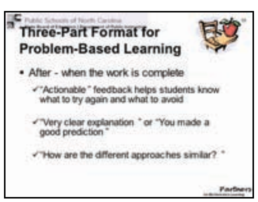
- *What am I being asked to do?*
- *What facts do I know?*
- *Is there any information I need to pay particular attention to?*
- *What kind of answer do I expect to get?*

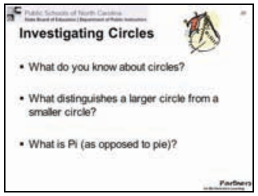


(slide 15) **Three-Part Format for Problem-Based Learning**
Allow time for the group to respond to the following questions.
How would you begin to develop these behaviors in your students?

- Teachers could model this behavior.
- After identifying students with this behavior, teachers could allow a student to model for the other students.

What other preparation might be needed?

	(slide 16) **Three-Part Format for Problem-Based Learning** Again, ask for a volunteer to read the bullets. ***<u>During – working on the task</u>*** • *Let go* – give students a chance to work without guidance • *Provide hints* – not solutions • *Listen actively* – "Tell me what you are doing?" – "Show me how you got that answer." – "Explain." • *Encourage testing of ideas* – "Why do you think that you might be right?" – "How can you check your solution?"	
	*(slide 17)* ***Three-Part Format for Problem-Based Learning*** ***<u>During – working on the task</u>*** • *Support students doing the thinking* • *Give pointers not step-by-step advice* • *Show me how you got that answer* • *How do you know that you are correct*	
	(slide 18) **Three-Part Format for Problem-Based Learning** ***<u>After – when the work is complete</u>*** • *Provide feedback* – what is correct and what needs to be revisited • *Use praise cautiously* – praise for one student may be negative feedback to another • *Engage the full class in discussion* – list answers from all of the groups, have one or more groups explain their solutions, allow students the opportunity to defend their answers, encourage students to ask other students questions	
	(slide 19) **Three-Part Format for Problem-Based Learning** ***<u>After – when the work is complete</u>*** • *"Actionable" feedback helps students know what to try again and what to avoid* • *"Very clear explanation" or "You made a good prediction"* • *"How are the different approaches similar?"*	



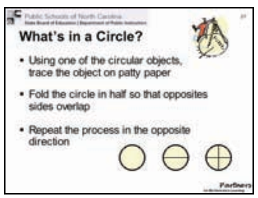
(slide 20) **Investigating Circles**

Be conscious in the following tasks of modeling the 3-part format for problem-based learning that you've just discussed.

Pi is the most well known irrational number and often the first irrational number that our students encounter within our mathematics curriculum.

Begin the following activity by discussing these questions with participants. Allow participants to briefly share their responses with a partner.

- *What do you know about circles?*
- *What distinguishes a smaller circle from a larger circle?*
- *What is Pi?*

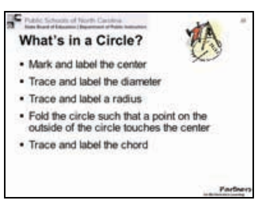


(slide 21) **What's in a circle?**

Students often misuse math vocabulary. It is important that teachers provide hands-on activities to help students identify and define mathematical terms. It is essential that students can identify parts of circle as they begin to use formulas for area and circumference of circles.

Give each participant a sheet of patty paper. Provide a variety of circular objects with diameters less than 15 cm. Ask a volunteer to read the following set of instructions aloud to the group. Ask if everyone is clear about the task and model if necessary.

- *Trace the circular object on the patty paper*
- *Fold the circle in half so that opposite sides overlap (create a diameter)*
- *Repeat the process in the opposite direction (create a center)*



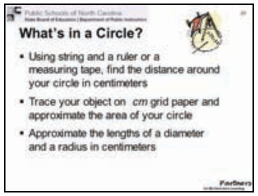
(slide 22) **What's in a circle?**

Students need to use specific vocabulary when communicating mathematical ideas.

NCTM PSSM states, students should "communicate their mathematical thinking coherently and clearly to peers, teachers, and others and use the language of mathematics to express mathematical ideas precisely."

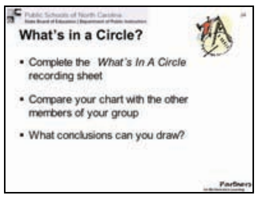
- *Mark and label the center*: the center of a circle is the same distance from all points on the circle and is the intersection of any two different diameters.
- *Trace and label the diameter*: the diameter is a line segment that passes through the center of a circle and has its endpoints on the circle.

- *Trace and label the radius*: the radius is a line segment with one endpoint at the center of a circle and the other endpoint on the circle.
- *Fold the circle such that one point on the outside of the circle touches the center.*
- *Trace and label the chord*: a chord is a line segment with endpoints on a circle.



(slide 23) **What's in a circle?**

- Ask participants to measure the distance around their circular object in centimeters using a string (and ruler) or measuring tape.
- Have participants trace their objects on cm grid paper and approximate the area of their circle (i.e. by counting square centimeters).
- Ask participants to approximate the lengths of a diameter and a radius in centimeters (i.e. counting the lengths of the squares).



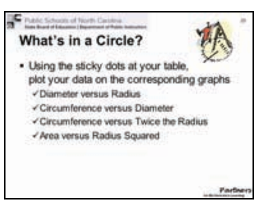
(slide 24) **What's in a circle?**

- Ask participants to find and complete the What's in a Circle? recording sheet (Module One, Handout Four)
- Ask participants to compare their charts with other members of their group.
- *What conclusions can you draw?*

$\frac{Diameter}{Radius} \approx 2 \quad \frac{Circumference}{Diameter} \approx 3 \quad \frac{Circumference}{2(Radius)} \approx 3 \quad \frac{Area}{Radius^2} \approx 3$

The larger the diameter/radius, the greater the area.
The larger the diameter/radius, the greater the circumference.
Radius is half the diameter.

Providing students with opportunities to discover relationships on their own has greater impact than teachers telling students formulas.



(slide 25) **What's in a circle?**

<u>Preparation ahead of time: The following chart paper with the axis labels mentioned below.</u>
Create 4 large graphs on chart grid paper representing the following relationships:

- Diameter (y) versus Radius (x)
- Circumference (y) versus Diameter (x)
- Circumference (y) versus Twice the Radius (x)
- Area (y) versus Radius Squared (x)

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<td>Ask participants to use the sticky dots to graph their data on the appropriate graphs.
Discuss independent and dependent variables: why are the axes labeled as they are?</td>
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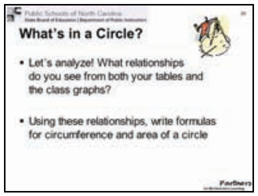
<td>
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<td>(slide 26) What's in a circle?
What relationships do you see from both your tables and the class graphs?

Graphs of circumference versus diameter and circumference versus twice the radius are the same.
The ratios of circumference to diameter and area to radius squared are the same. Therefore one should recognize that the formulas calculating circumference and area using diameter and radius will contain a constant approximately equal to 3.
Rate of change is approximately 3 for the circumference/diameter, circumference/twice the radius, and area/radius squared graphs. (For every 1 cm increase in diameter, the circumference increases by 3 cm.)
Rate of change is 2 for the diameter/radius relationship. (For every 1 cm increase in radius, there is a 2 cm increase in the diameter.)

Using these relationships, develop formulas for circumference and area of a circle.

The exact ratio of the circumference to diameter is an irrational number close to 3.14 and is represented by the Greek letter, π.
π is exact, but 3.14 and $\frac{22}{7}$ are approximations. How might students compare 3.14 and $\frac{22}{7}$? Placement on a number line
<u>Circumference of a Circle</u>: $C = \pi d$ or $C = 2\pi r$
<u>Area of a Circle</u>: $A = \pi r^2$

Literature Piece: SIR CUMFERENCE Series by Cindy Neuschwander
Show participants various books available.</td>
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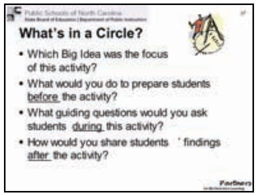


(slide 27) **What's in a circle?**

Which Big Idea was the focus of this activity?

- Encourage participants to support/explain their choices
- Measurement: Formulas are derived from the measures of the attributes and relationships of 2-D and 3-D figures
- Geometry: Two and three dimensional figures can be classified and distinguished by their properties or attributes
- Geometry: Mathematical equations can be used to describe geometric relationships within and between geometric figures
- Geometry: Formulas that describe attributes of geometric figures can be utilized for indirect measurements and problem solving
- Participants may list other Big Ideas; remember not to discredit other options.

What would you do to prepare students before this activity?

- Experiences finding area on cm grid paper
- Experiences using rulers and measuring tapes
- Experiences with ratios

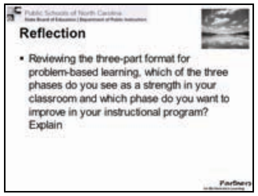
What guiding questions would you ask students during this activity?

- What do you know about the radius and diameter of a circle and how are they related?
- Can a diameter also be described as a chord? **Yes**
- Are all diameters chords? **Yes** Are all chords diameters? **No**
- What strategy did you use to determine the area of your circle?
- Do the relationships hold true for all circles?

How would you share students' findings after the activity?

- Create a class chart for students to display their data
- Monitor group discussions and listen for unique strategies and ask these groups to share at the end

Extension Activity: Use a graphing calculator or spreadsheet for graphing the circle data.

	(slide 28) **Reflection** Have participants write their reflective responses on a page in their handout packet. *Reviewing the three-part format for problem-based learning, which of the three phases do you see as a strength in your classroom and which phase do you want to improve in your instructional program? Explain.* Remind participants that they should bring their copies of the Big ideas and the 2009 Standard Course of Study to each Partners professional development session.	
	(slides 29-32) **Credits for project and closing slides**	