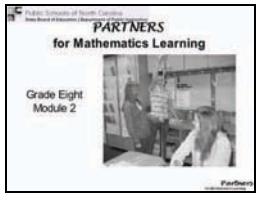
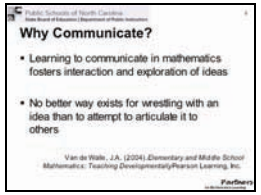
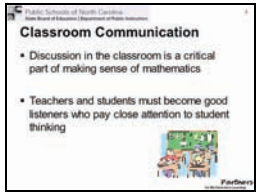
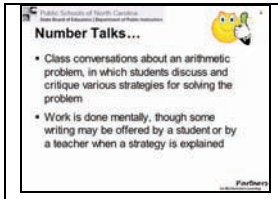
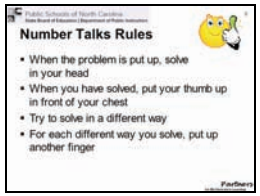
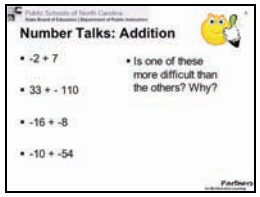
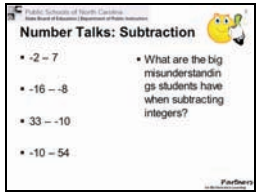
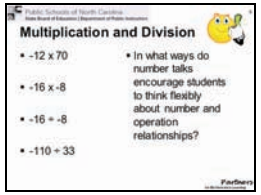
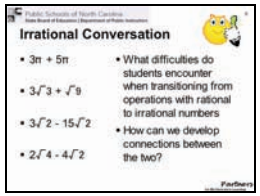
General Materials and Supplies:	White board, chart paper, or chalkboard	Markers or chalk
Eraser	Envelopes	Handouts #5-9

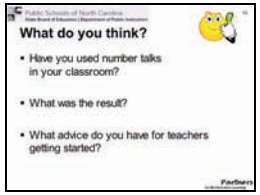
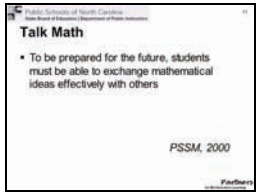
Slide	Tasks/Activity	Personal Notes
	(slide 1) **Grade Eight, Module 2** Module 2 focuses on communication. Communication was included as an NCTM *Process Standard* as a necessary piece of mathematics instruction that needs to be considered in planning, implementation, and assessment of mathematical tasks.	
	(slide 2) **Why Communicate?** • *Learning to communicate in mathematics fosters interaction and exploration of ideas.* • *No better way exists for wrestling with an idea than to attempt to articulate it to others.* Ask participants "Why is communication so important?" • Students live in an active verbal environment. • Mathematical expression, therefore, is part of the process and not an end in itself. Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc.	
	(slide 3) **Classroom Communication** • *Discussion in the classroom is a critical part of making sense of mathematics.* • *Teachers and students must become good listeners who pay close attention to student thinking.* Communication in the mathematics classroom helps students solidify their understanding and helps teachers learn about their students.	
	(slide 4) **Number Talks** • *A class conversation about an arithmetic problem, in which students discuss and critique various strategies for solving the problem.* • *The work is done mentally, though some writing may be offered by a student or by a teacher when a strategy is explained.*	

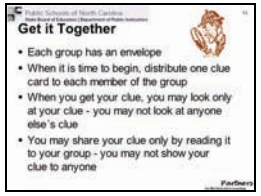
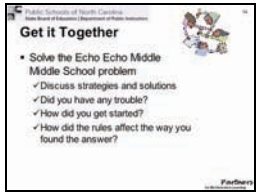
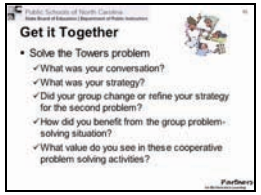
	These activities were modeled after "Number Talk" activities developed by Dr. Ruth Parker.	
	(slide 5) **Number Talks Rules** • *When the problem is put up, solve in your head.* • *When you have solved, put your thumb up in front of your chest.* • *Try to solve in a different way. For each different way you solve, put up another finger.* Have a participant read the rules of Number Talks out loud to the group.	
	(slide 6) **Number Talks: Addition** MATERIALS: white board or chalkboard or chart paper, markers, eraser There are four examples. Choose two to use with your group. Write the teachers' solutions in a place visible to all teachers such as chart paper a whiteboard or chalkboard in the room. Possible strategies include: *-2 + 7* • 7 – 2 • Start at -2 and count up 7: -2, -1, 0, 1, 2, 3, 4, 5 • Since 7 = 2 + 5, -2 + 7 = -2 + (2 + 5) = (-2 + 2) + 5 = 0 + 5 = 5 *33 + -110* • Subtract using the common subtraction algorithm: subtracting right to left. • 11 – 3 = 8, therefore 110 – 30 = 80 and then 80 – 3 = 77. Since -110 is greater absolute value, the answer is -77. • Count up from 33: 33 + 7 = 40, 40 + 70 = 110, so the difference between 33 and 110 is 77. Since -110 is greater absolute value, the answer is -77. *-16 + -8* • Start at -16 on the number line and count down 8 to -24.	

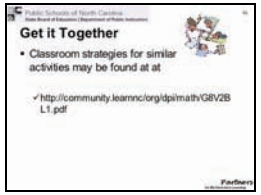
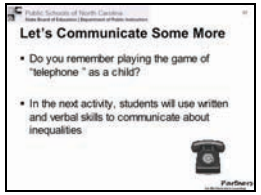
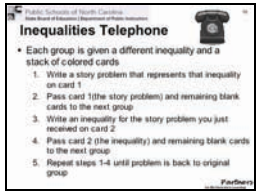
	• Add the absolute value of 16 and 8, 24. Since the signs were negative, the 24 is negative. *-10 + -54* • Start at -10 on the number line and count down 54 to – 64. • Add 10 to 54. Since I know that the sum of a negative number and another negative number is always negative, my answer is -64. • 1 + 5 = 6 so 10 + 54 = 64. And -10 + -54 = -64 Ask participants to respond to the questions on the slide. *Is one of these more difficult than the others?* *Why?*	
	(slide 7) **Number Talks: Subtraction** There are four examples. Choose two to use with your group. Write the teachers' solutions in a place visible to all teachers such as chart paper a whiteboard or chalkboard in the room. Possible strategies include: *-2 – 7* • Start at -2 and count down 7 to -9. • Taking 7 from -2 means the number will be 7 units smaller than -2, or -9. • Think of it in terms of money. If someone owes $2 and then owes $7, that person owes $9 in all. Since "owes" represents a negative value, -2 – 7 = -9. *-16 - -8* • Taking -8 from -16 results in a quantity of -8. • Distribute the minus, and rewrite the equivalent equation: -16 + 8. Adding 8 to -16 yields -8. *33 - -10* • To subtract -10 from 33, rename 33 as -10 +10 + 33. The equation can be rewritten as -10 + 10 + 33 – -10 = 10 + 33 = 43 • Distribute the minus, and rewrite the equivalent equation: 33 + 10. The answer is 43.	

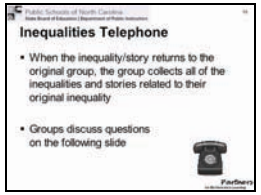
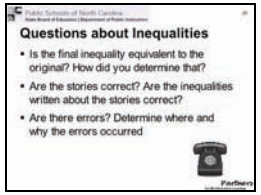
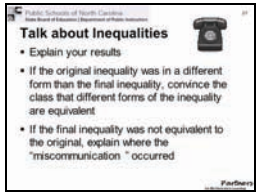
	-10 - 54 • To take 54 from -10, count down from -10 to -64. • Some students may count down 10 from -54 to -64. • If one owes 10 and then overdraws 54, the amount owed is 64. Ask participants to respond to the question on the slide. *What are the big misunderstandings students have when subtracting integers?*	
	(slide 8) **Multiplication & Division** There are four examples. Choose two to use with your group. Write the teachers' solutions in a place visible to all teachers such as chart paper a whiteboard or chalkboard in the room. Possible strategies include: *-12 x 70* • 12 x 7 = 84. Since it is not 7, but 70, we know 12 x 70 = 840. Since the first factor is -12, then we know the product of -12 and 70 is -840. • 10 x 70 = 700 and 2 x 70 = 140. 700 + 140 = 840. Since one factor is negative the answer is -840. *-16 x -8* • Use the traditional algorithm, 8 x 6 = 48. 8 x 10 = 80. 80 + 48 = 128. • 20 x 8 = 160. 4 x 8 = 32. 160 – 32 = 128. *-16 / -8* • We know -8 x 2 = -16. The quotient is 2. • There are two groups of -8 in -16. The quotient is 2. *-110/33* • It can be simplified to -10/3. • The answer is a little less than -3. • 33 x 3 = 99. 110 – 99 = 11. So the quotient is -3, remainder 11. Have participants respond to the question on the slide.	

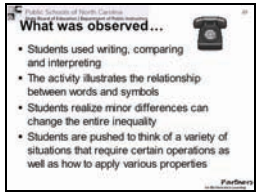
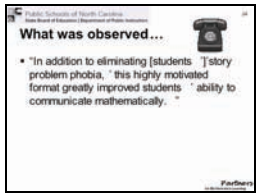
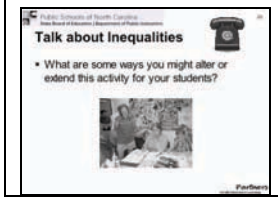
	In what ways do Number Talks encourage students to think flexibly about number and operation relationships? Number Talks challenge students to develop more than one strategy, which can only be done by applying number and operation relationships. During a Number Talk everyone is focused on number and operation relationships, not just getting an answer.	
	(slide 9) **Irrational Conversation** There are four examples. Choose two to use with your group. Write the teachers' solutions in a place visible to all teachers such as chart paper a whiteboard or chalkboard in the room. Possible strategies include: *3π + 5π* • We can think of pi as a unit: 3 dollars + 5 dollars = 8 dollars. Or 3 apple pies added to 5 apple pies gives a total of 8 apple pies. • Using the distributive property $(3 + 5)\pi = 8\pi$ *3√3 + √9* • Simplify √9 to 3. The equivalent expression is 3√3 + 3. • An approximate answer could be reached through estimation. Since √*3* is between √*1* and √*4*, it could be estimated as 1.5. 3 x 1.5 = 4.5. 4.5 + 3 = 7.5. *3√2 - 15√2* • Here, we can think of √*2* as a unit: 3 squares – 15 squares = -12 squares. Or -12√2 • 3 – 15 = -12. The answer is -12√2 2√4 - 4√2 • √4 = 2. 2 x 2 = 4. 4 - 4√2. • A student may estimate the answer. Since √4 = 2, 2 x 2 = 4. Since √2 is between √1 and √4, it could be approximated with a number greater than 1 and less than 2, such as 1.5. 4 x 1.5 is 6. 4 – 6 is -2. The answer is close to -2. Ask participants to respond to the question on the slide. *What difficulties do students encounter when transitioning from operations with rational to*	

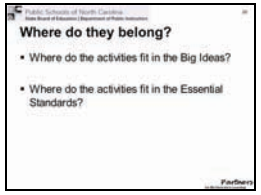
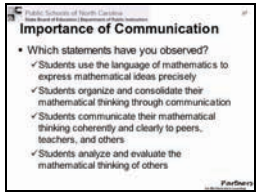
	irrational numbers? *How can we develop connections between the two?* Using the exact value of an irrational number allows students to operate with irrational numbers in manners similar to operating with variables.	
	(slide 10) **What do you think?** *Have you used number talks in your classroom?* *What was the result?* *What advice do you have for teachers getting started?* If participants have not yet used number talks in their classroom, discuss the ways their students would benefit from this activity.	
	(slide 11) **Talk Math** *To be prepared for the future, students must be able to exchange mathematical ideas effectively with others.* *PSSM, 2000* Ask participants to discuss the slide's quote with a neighbor. Do you agree with the quote? Why or why not?	
	(slide 12) **Student Conversation = Engagement** Tell participants: "When the teacher is the only one talking, many students stop paying attention. When students are in a group, they are almost always more likely to be engaged." Ask participants if they have observed this. Next we are going to do an activity that gives students an opportunity to work together to solve problems. Pass out the envelopes for the Echo Echo Middle School Problem. Equals (1989). *Get it Together: Math Problems for Groups*	

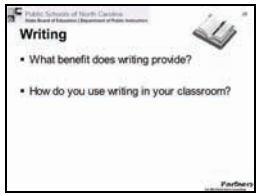
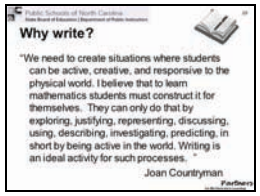
	(slide 13) **Get it Together** MATERIALS: Two envelopes per group: one envelope contains one set of Echo Echo cards (handout #5) and the other envelope contains one set of the Towers problem cards (handout #6). • *Each group has an envelope.* • *When it is time to begin, distribute one clue card to each member of the group.* If it is a small group, some members may have more than one clue. • *When you get your clue, you may look only at your clue. You may not look at anyone else's clue.* • *You may share your clue only by reading it to your group. You may not show your clue to anyone.* Equals (1989). *Get it Together: Math Problems for Groups*	
	(slide 14) **Get it Together** *Solve the Echo Echo Middle Middle School problem* ▪ *Discuss strategies and solutions* ▪ *Did you have any trouble?* ▪ *How did you get started?* ▪ *How did the rules affect the way you found the answer?* Give participants an opportunity to complete the activity. Discuss participants' strategies and solutions. Ask groups to answer the questions on the slide.	
	(slide 15) **Get it Together** Pass out envelopes for the Towers problem. *Solve the Towers problem* ▪ *What was your conversation?* ▪ *What was your strategy?* ▪ *Did your group change or refine your strategy for the second problem?* ▪ *How did you benefit from the group problem-solving situation?* ▪ *What value do you see in these cooperative problem solving activities?* All students are engaged. Students observe multiple problem- solving strategies.	

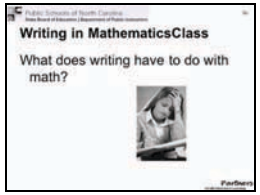
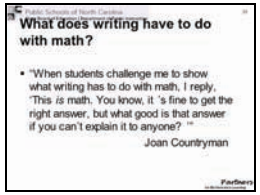
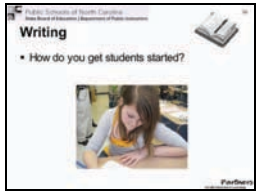
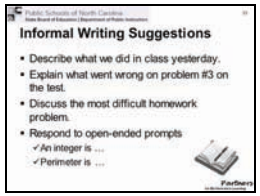
	Students experience success. Teachers can spend more time observing students. Teachers can assess student ability. Teachers have more time for individual conversations.	
	(slide 16) **Get it Together** Classroom strategies blackline masters for similar activities can be found online at http://community.learnnc.org/dpi/math/G8V2BL1.pdf	
	(slide 17) **Let's Communicate Some More** *Do you remember playing the game of "telephone" as a child?* Whisper a phrase to one of the teachers and pass it around the room such as "North Carolina students will excel in mathematics". What did the last person hear? *In the next activity, students will use written and verbal skills to communicate about inequalities.* Inequalities Telephone Adapted from Bay, J and Ragan, G., *Improving Students' Mathematical Communication and Connections Using the Classic Game of "Telephone",* Mathematics Teaching in the Middle School, Vol. 5, Number 8, April 2000, pp 486-489.	
	(slide 18) **Inequalities Telephone** MATERIALS: different colored cards, inequality strips. Each group will have a different colored stack of cards and a different inequality. The number of blank cards should equal the number of groups. (For example, if your class has 6 small groups, each group should have 6 blank cards and the inequality card of the same color.) *Each group is given a different inequality and a stack of colored cards.* 1. *Write a story problem that represents that inequality on card 1.* 2. *Pass card 1(the story problem) and remaining blank cards to the next group.*	

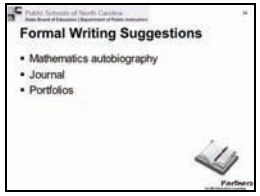
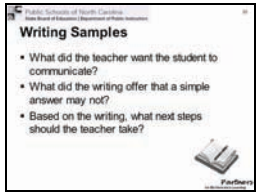
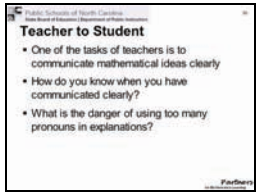
	3. *Write an inequality for the story problem you just received on card 2.* 4. *Pass card 2 (the inequality) and remaining blank cards to the next group.* 5. *Repeat steps 1-4 until problem is back to original group.* Allow approximately 5 minutes to write the story and 2 minutes to write the inequality. (When teachers use this activity in the classroom, they may need to allow as much as twice the time.) This is not about finding solutions. The focus is communicating mathematical ideas.	
	(slide 19) **Inequalities Telephone** ▪ *When the inequality/story returns to the original group, the group collects all of the inequalities and stories related to their original inequality.* Each group will collect all of the cards that are the same color as their original inequality strip. ▪ *Groups discuss questions on the following slide.*	
	(slide 20) **Questions about Inequalities** Then each group reviews the inequalities and stories written about the inequalities and answers the following questions within their group. *1) Is the final inequality equivalent to the original? How did you determine that?* *2) Are the stories correct? Are the inequalities written about the stories correct?* *3) Are there errors? Determine where and why the errors occurred.*	
	(slide 21) **Talk about Inequalities** ▪ *Explain your results.* ▪ *If the original inequality was in a different form than the final inequality, convince the class that different forms of the inequality are equivalent.* ▪ *If the final inequality was not equivalent to the original, explain where the "miscommunication" occurred.* Groups discuss their results with the whole class.	

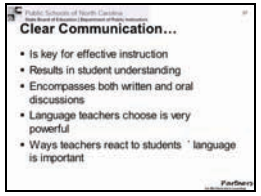
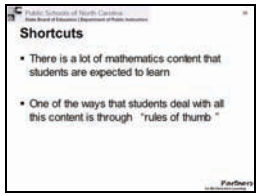
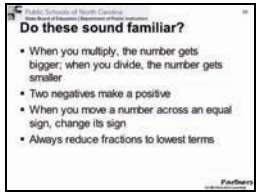
	(slide 22) **Talk about Inequalities** *What benefit does this kind of activity offer your students?* Answers may include: It gets students thinking about what the inequality means. It is a way for students to practice and stay engaged. It is a way for the teacher to informally assess student understanding of inequalities.	
	(slide 23) **What was observed…** This activity was adapted from an article in the April 2000 issue of Mathematics Teaching in the Middle School called, *Improving Students' Mathematical Communication and Connections Using the Classic Game of "Telephone"* (handout 7). The authors observed that ▪ *Students used writing, comparing and interpreting.* ▪ *The activity illustrated the relationship between words and symbols.* ▪ *Students realized minor differences can change the entire inequality.* ▪ *Students were pushed to think of a variety of situations that require certain operations as well as how to apply various properties.*	
	(slide 24) **What was observed…** The authors further concluded *In addition to eliminating [students'] "story problem phobia," this highly motivated format greatly improved students' ability to communicate mathematically.*	
	(slide 25) **Talk about Inequalities** *What are some ways you might alter or extend this activity for your students?* Suggestions may include: One group could graph the inequality, the next group could write the inequality, and the next group could write a story.	

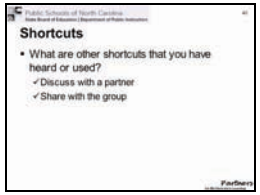
	It could be done several times throughout the year, starting with one variable equations then inequalities and moving into two variable equations and inequalities.	
	(slide 26) **Where do they belong?** Participants may come up with any of the following. Recall that participants were given copies of the Big Ideas and the Essential Standards during the first module and asked to bring them to later meetings. It may be helpful to have extra copies at hand. *Where do the activities fit in the Big Ideas?* • A variety of representations (including tables, charts, graphs, number lines, expressions, equations, and inequalities) can be used to illustrate mathematical relationships, to model mathematical situations, or to describe and generalize patterns • Expanded understanding and use of classes of numbers increases students' abilities to describe situations and solve problems • Fluency with different types of reasoning (quantitative, additive, multiplicative, proportional) is necessary for mathematical development • Fluency (accuracy, efficiency, flexibility) using operations with rational numbers becomes solidified in the middle grades *Where do the activities fit in the Essential Standards?*	
	(slide 27) **Importance of communication** MATERIALS: Communication Statement cards (handout 5). *Which statements have you observed?* ▪ *Students use the language of mathematics to express mathematical ideas precisely.* ▪ *Students organize and consolidate their mathematical thinking through communication.* ▪ *Students communicate their mathematical thinking coherently and clearly to peers, teachers, and others.* ▪ *Students analyze and evaluate the mathematical thinking and strategies of others.* Participants should be in groups of at least 2 people, depending on the number of participants. Pass out one statement card to each group. Ask groups to discuss the statement on the card and to	

	provide at least one example to support the statement. If groups finish before time is called, they may discuss the other statements on the board. Take a few minutes for groups to share their examples.	
	(slide 28) **Writing** Please state, "Just as oral communication is an important part of the learning process, written communication plays a major role as well." *What benefit does writing provide?* Encourage teachers to think about the positive aspects of writing. Writing offers a way to learn about students' thinking. *How do you use writing in your classroom?* Let's look into ways to use writing to help students and teachers.	
	(slide 29) **Why write?** Ask a participant to read the quote. *"We need to create situations where students can be active, creative, and responsive to the physical world. I believe that to learn mathematics students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short by being active in the world. Writing is an ideal activity for such processes."* This quote from Joan Countryman discusses the process of writing. Countryman, Joan. (1992) *Writing to Learn Mathematics*. Heinemann Educational Books Inc.	

	(slide 30) **Writing in Mathematics Class** *What does writing have to do with math?* Ask participants what they think about writing and mathematics. They may answer that writing shows that the student understands the mathematics. Writing helps solidify the student's own knowledge and understanding. Writing can show where a student misunderstands or is confused.	
	(slide 31) **What does writing have to do with math?** Ask a participant to read the quote. *"When students challenge me to show what writing has to do with math, I reply, 'This is math. You know, it's fine to get the right answer, but what good is that answer if you can't explain it to anyone?'"* Countryman, Joan. (1992) *Writing to Learn Mathematics*. Heinemann Educational Books Inc.	
	(slide 32) **Writing** *How do you get students started?* Ask participants to think about the question. After a few moments, ask them to discuss their answers with a partner. Then after a minute or so, ask pairs to share their ideas.	
	(slide 33) **Informal writing suggestions** • *Describe what we did in class yesterday.* • *Explain what went wrong on problem #3 on the test.* • *Discuss the most difficult homework problem.* • *Respond to open-ended prompts* ✓ *An integer is …* ✓ *Perimeter is …* Some suggestions for beginning writing in the mathematics classroom are included here. Each of these suggestions offers insight into students' thinking and understanding of mathematics.	

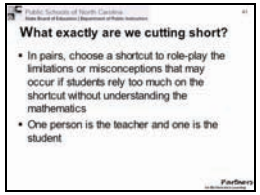
	(slide 34) **Formal writing suggestions** • *Mathematics autobiography* • *Journal* • *Portfolios* An autobiography gives an idea of the student's attitude toward mathematics. A journal can be used throughout the year.	
	(slide 35) **Writing Samples** MATERIALS: Handout #6, writing samples • *What did the teacher want the student to communicate?* • *What did the writing offer that a simple answer may not?* • *Based on the writing, what next steps should the teacher take?* Pass out Handout #6. Share and discuss student-writing samples as time permits. You may want to assign a subset of the samples to each group for analysis.	
	(slide 36) **Teacher to Student** *One of the tasks of teachers is to communicate mathematical ideas clearly.* Ask the teachers, "What does 'communicate mathematical ideas clearly' mean?" If we want our students to communicate mathematical ideas clearly, we must model that for them. *How do you know when you have communicated clearly?* For example, students are able to "say back" what you've said correctly, but in their own words. *What is the danger of using too many pronouns in explanations?* Students may not be clear about what a pronoun refers to and this avoids the use of mathematical language. Bright, G. and Joyner, J. (2004). *Dynamic Classroom Assessment: Linking Mathematical Understanding to Instruction in Middle Grades and High School,* ETA/Cuisenaire.	

	(slide 37) **Clear Communication** ▪ *Key for effective instruction* ▪ *Results in student understanding* ▪ *Encompasses both written and oral discussions* ▪ *Language teachers choose is very powerful* ▪ *Ways teachers react to students' language is important* Bright, G. and Joyner, J. (2004). *Dynamic Classroom Assessment: Linking Mathematical Understanding to Instruction in Middle Grades and High School,* ETA/Cuisenaire.	
	(slide 38) **Shortcuts** *There is a lot of mathematics content that students are expected to learn.* *One of the ways that students deal with all this content is through shortcuts or "rules of thumb."* Sometimes teachers or mathematics textbooks provide the shortcuts, and sometimes students create them spontaneously. Shortcuts may be limiting; that is, they may not accurately reflect the complexity of the underlying mathematical ideas. Bright, G. and Joyner, J. (2004). *Dynamic Classroom Assessment: Linking Mathematical Understanding to Instruction in Middle Grades and High School,* ETA/Cuisenaire	
	(slide 39) **Do these sound familiar?** • *When you multiply, the number gets bigger; when you divide, the number gets smaller.* • *Two negatives make a positive.* • *When you move a number across an equals sign, change its sign.* • *Always reduce fractions to lowest terms.* The shortcuts on this slide have been provided to begin the conversation about shortcuts and so that your group has enough shortcuts to do the activity that follows.	



(slide 40) **Shortcuts**
MATERIALS: White board or chalkboard or chart paper, markers
What are other shortcuts that you have heard or have used?
Discuss with a partner
Share with the group

Create a list from the teachers' suggestions. Write it in a place that is visible to everyone.



(slide 41) **What exactly are we cutting short?**
In pairs, choose a shortcut to role-play the limitations or misconceptions that may occur if students rely too much on the shortcut without understanding the mathematics.
One person is the teacher and one is the student.

Shortcuts can be from the teacher-created list on the chart paper or from the slide.
If teachers need help finding flaws in the shortcuts provided on the slide, use the discussion points below.

Discussion Points:

- *When you multiply, the number gets bigger; when you divide, the number gets smaller.*

This idea is true for whole numbers but fails for fractions/decimals less than or equal to 1.

- *Two negatives make a positive.*

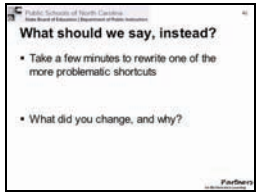
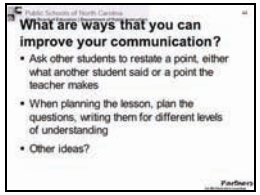
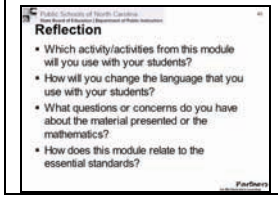
This rule is often introduced when students learn to multiply signed numbers. It fails for many other operations, such as addition and subtraction.

- *When you move a number across an equals sign, change its sign.*

This might be taught when a student is learning to solve equations by adding or subtracting a quantity from both sides. However, the rule does not apply when multiplying both sides of an equation by a constant, as in solving the equation $x/2 = 3$ by multiplying by two. Some students may "see" this operation as "moving the 2 across the equals sign" and want to change its sign.

- *Always reduce fractions to lowest terms.*

Sometimes it is useful *not* to rename a fraction; sometimes renaming fractions in lowest terms is inefficient. For example, if a problem requires the computation of several fractions each having a

	denominator of 50, and if a later stage of the problem will require the fractions to be added, then it would be best to retain the common denominator instead of renaming the fractions in lowest terms. Remember that the value represented by a fraction is not dependent on whether the fraction is written in lowest terms.	
	(slide 42) **What should we say, instead?** *Take a few minutes to rewrite one of the more problematic shortcuts.* *What did you change, and why?* Circulate through the group making sure different shortcuts are chosen. If every group chooses the same shortcut, challenge each group with another. After about 5 minutes, ask for volunteers to share their answers.	
	(slide 43) **What are ways that you can improve your communication?** • *Tape record or videotape your class:* • *Analyze the tape* o *How clear were your directions?* o *How many pronouns do you use?* o *Did you misunderstand a student?* o *Did you provide adequate "think time"?*	
	(slide 44) **What are ways that you can improve your communication?** • *Ask other students to restate a point, either that another student says or a point that the teacher makes.* • *When planning the lesson, plan the questions. Write them at different levels of understanding.* • *Other ideas?*	
	(slide 45) **Reflection** *Which activity/activities from this module will you use with your students?* *How will you change the language that you use with your students?* *What questions or concerns do you have about the material presented or the mathematics?* *How does this module relate to the Essential Standards?*	

	Reflections can be either written down for personal use or to share with the group, or a group discussion may occur.	
	(slide 46-48) **Credit slides**	