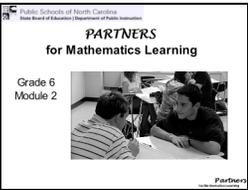
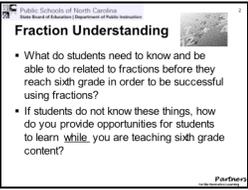
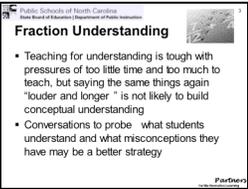
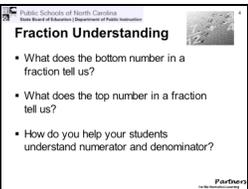
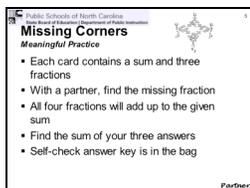


General Materials and Supplies:

Laptop, Projector, Power Cord	Music Match Cards	Sets of Ratio Cards	Standard paper clips (30mm)
Speakers	Sets of Multiplication Magic Cards	Masking Tape	Jumbo paper clips (45mm)
Sets of Missing Corners Cards	Small Sticky Notes	Music CD	
<i>Missing Corners</i> Recording Sheet	<i>Proportional Links</i> Handout	<i>Multiplication Magic</i> Handout	

Slide	Tasks/Activity	Personal Notes
	<p>(slide 1) Module Two This module focuses on Number and Operations with an emphasis on fractions. You might bring a few extra copies of the Big Ideas and 2009 Standard Course of Study in case participants forget to bring their copies.</p>	
	<p>(slide 2) Fraction Understanding <i>What do students need to know and be able to do related to fractions before they get to 6th grade, in order to be successful using fractions?</i> Ask participants to brainstorm a list at their tables that they will share with the group. A list might include the following:</p> <ul style="list-style-type: none"> • What the numerator and denominator represent • Fractions representing a Part-to-Whole relationship, with equal portions • Fractions of regions don't have to be congruent • At the symbolic level and within calculations the "whole" is always assumed to be the same • Fractions can be named in multiple ways, equivalent fractions, mixed numbers, decimals • Whole numbers can be represented as fractions • Fraction notation is one representation of division • Estimation skills with fractions <p><i>If your students do not know these things how do you provide opportunities for students to learn while you are teaching sixth grade content?</i> Ask participants to brainstorm a list at their tables that they will share with the group. A list might include the following:</p> <ul style="list-style-type: none"> • Talk with students to identify misconceptions or misunderstandings (conceptual misunderstanding or procedural misunderstanding) 	

	<ul style="list-style-type: none"> • Provide multiple representations of fractions and of operations with fractions (pictures or manipulatives along with notation) • Make connections to strategies for doing operations with whole numbers- this may entail collaborating with 4th and 5th grade teachers to discover those whole number strategies • Allow students to create and explore their own strategies for operating with fractions along with showing them “standard” algorithms 	
	<p>(slide 3) Fraction Understanding There is a logic behind students' answers. When students make mistakes there is almost always a misconception or incomplete understanding that leads to that wrong answer.</p> <ul style="list-style-type: none"> • <i>Teaching for understanding is tough with pressures of too little time and too much to teach, but saying the same things “louder” and “longer” is not likely to build conceptual understanding</i> • <i>Conversations to probe what students understand and what misconceptions they have may be a better strategy</i> 	
	<p>(slide 4) Fraction Understanding <i>What does the bottom number in the fraction tell us?</i></p> <ul style="list-style-type: none"> • Tells what is being counted • A denomination is a name of a class or type of thing • The bottom number in the fraction is the <i>denominator</i> <p><i>What does the top number in the fraction tell us?</i></p> <ul style="list-style-type: none"> • The top number counts • Enumeration is the process of counting • The top number in the fraction is the <i>numerator</i> <p><i>Teaching Student Centered Mathematics Grades 5 – 8</i> <i>John A. Van de Walle and LouAnn H. Lovin</i></p> <p><i>How do you help students understand numerator and denominator?</i></p> <ul style="list-style-type: none"> • Use the correct terminology all the time • Numerator is the multiplier and the denominator is the divisor • Use pictures and representations • Answers will vary 	

**(slide 5) Missing Corners**

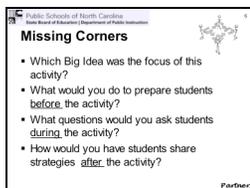
Knowing that operations with fractions is challenging for many students, using activities that engage students is important for the development of such difficult concepts.

Preparation Ahead of Workshop

Cut out each set of the Missing Corner Puzzles (Module Two, Handout One). There are 8 sets and three puzzles to a set. Place each set in a baggie or envelope. You may or may not want to include the self-check answer key with the set. Copy the sets on cardstock and ask participants to use the recording sheet (Module Two, Handout One), or laminate the sets and have participants use dry erase markers to record answers.

Participants will work with a partner to complete this activity.

- Give each pair a set of *Missing Corner* cards.
- With a partner, find the missing fraction. All four fractions should add to the given sum.
- Once all three answers have been found, find the sum of the three results. Answer key is in the bag.

**(slide 6) Missing Corners**

Which Big Idea was the focus of this activity?

Fluency (accuracy, efficiency, flexibility) using operations with rational numbers becomes solidified in the middle grades

What would you (the teacher) do to prepare students before this activity?

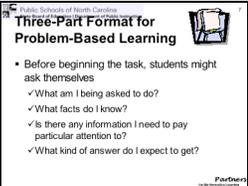
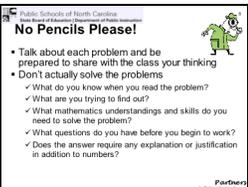
- Estimate with fractions
- Grouping fractions to make a whole
- Like denominators
- Answers will vary

What questions would you ask students during the activity?

- What strategy did you use to find the missing corner?
- Did you always have to find a common denominator to find the missing corner?
- Is there more than one way to find the missing corner?

Things to look (assessment) for during the activity:

- How are the pair of students working together and communicating mathematically?

	<ul style="list-style-type: none"> • What processes are the students using to find the missing part and are the processes reasonable? <p><i>How would you have students share strategies <u>after</u> the activity?</i></p> <ul style="list-style-type: none"> • Note student strategies during observation of the activity, ask particular students to show their strategies on the board • Answers will vary <p><i>Was there a set that was more challenging than the others? Explain.</i></p> <ul style="list-style-type: none"> • Sets that had different denominators • Answers will vary 	
 <p>Public Schools of North Carolina Three-Part Format for Problem-Based Learning</p> <ul style="list-style-type: none"> • Before beginning the task, students might ask themselves <ul style="list-style-type: none"> ✓What am I being asked to do? ✓What facts do I know? ✓Is there any information I need to pay particular attention to? ✓What kind of answer do I expect to get? <p>Partners</p>	<p>(slide 7) Three-Part Format for Problem-Based Learning</p> <p>Recall from the first module...</p> <p>Research from <i>Teaching Student Centered Mathematics Grades 5 – 8</i> John A. Van de Walle and LouAnn H. Lovin</p> <p><i>Before beginning the task, <u>students</u> might ask themselves</i></p> <ul style="list-style-type: none"> • <i>What am I being asked to do?</i> • <i>What facts do I know?</i> • <i>Is there any information I need to pay particular attention to?</i> • <i>What kind of answer do I expect to get?</i> 	
 <p>Public Schools of North Carolina No Pencils Please!</p> <ul style="list-style-type: none"> • Talk about each problem and be prepared to share with the class your thinking • Don't actually solve the problems <ul style="list-style-type: none"> ✓What do you know when you read the problem? ✓What are you trying to find out? ✓What mathematics understandings and skills do you need to solve the problem? ✓What questions do you have before you begin to work? ✓Does the answer require any explanation or justification in addition to numbers? <p>Partners</p>	<p>(slide 8) No Pencils Please</p> <p>Have participants find the No Pencils Please handout in their materials (Module Two, Handout Two). Ask them to read the instructions individually, then ask the group for questions before beginning the activity.</p> <ul style="list-style-type: none"> • <i>Talk about each problems and be prepared to share with the class what you are thinking</i> • <i>Don't actually solve the problems</i> <p>Write the problems on chart paper and have participants record strategies for solving. After participants have finished, use a gallery walk to look at the different strategies. Choose different tables to respond to the whole group about the problems, answering the questions on the handout.</p>	

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What is a Ratio?

- A **ratio** is a comparison of two quantities or measures
- Comparing same quantities or measures
 - ✓ Part to Whole
 - ✓ Part to Part
- Comparing different measures
 - ✓ Rates



Partners

(slide 9) **What is a ratio?**

- A comparison of two quantities
- Something to something
- Answers will vary

A ratio is a comparison of two quantities or measures

- *Comparing same quantities or measures*
 1. *Part to Whole* (8 girls to 20 students)
 2. *Part to Part* (8 girls to 12 boys)
- *Comparing different measures*
 1. *Rates* (60 miles to 1 hour)

It's important for students to see a ratio as a relationship between values and not two separate parts. Ratios are the beginning of proportional reasoning. Proportions involve multiplicative reasoning rather than additive reasoning. Equal ratios are the result of multiplication not addition. Ask participants for examples of each category of ratio. Then ask participants for other definitions of ratio they have used in the past.

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Ratios to the Rescue

- Separate the ratio cards into the three categories
 - ✓ Part to Whole
 - ✓ Part to Part
 - ✓ Rates



Partners

(slide 10) **Ratios to the Rescue**

Give each pair of participants a set of *Ratios to the Rescue* (Module Two, Handout Three) cards.

- Ask participants to separate the cards into the three categories
 1. *Part to Part*
 2. *Part to Whole*
 3. *Rates*

Ask participants to share their groupings.

Did all the groups agree? Were any of the cards hard to classify? If so, why?

Public Schools of North Carolina
 Department of Instruction, Assessment and Accountability

Ratios to the Rescue 

Concept Check

- If you know that in a classroom the ratio of boys to girls is 2/3, what would be some reasonable values for the total number of students in the class?
- Explain your reasoning
- Why do some students struggle with this type of problem?

Partners

(slide 11) **Concept Check**

Have participants think about this question on their own and then turn to a partner to discuss. Take a few minutes for participants to share their ideas.

If you know that in a classroom the ratio of boys to girls is 2/3, what would be some reasonable values for the total number of students in the class?

Explain your reasoning.

Why do some students struggle with this type of problem?

This ratio represents a part-to-part relationship. One possible answer would be 20 students- 8 boys and 12 girls.

This is a good time to discuss the different notations that are used with ratio. The two notations are the colon notation and the fraction notation. The colon notation is often used with part to part comparisons and the fraction notation is used with part to whole comparisons.

It is very important for teachers and students to include units when writing these relationships. If units are not used, the conceptual differences between ratio and fraction are lost, particularly in the part-to-part examples.

Example:

$\frac{4 \text{ adults}}{10 \text{ children}}$ without the units, one might see $\frac{4}{10}$ as 4 out of a total of 10

instead of 4 out of a total of 14 people

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 Department of Public Instruction

Ratios to the Rescue 

- How does this activity build a foundation for proportional reasoning?
- What essential standard does this address?
- After this activity, how can you assess student understanding of ratio?

Partners

(slide 12) **Ratios to the Rescue**

How does this activity build a foundation for proportional reasoning?

- Develops multiplicative thinking
- Recognize differences between the three types of ratios
- Develops foundation for probability (odds)
- Answers will vary

What Essential Standard does this address?

Answers may vary. Allow time to look through the Standards.

After this activity how can you assess student understanding of ratio?

- Did the students have reasonable explanations of their first sort?
- Does each student understand the different ratio relationships?
- Can the students determine the parts of the ratio and how it can be used?
- Can the students create their own ratio relationships?

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 Department of Public Instruction

Proportional Links 

- Sasha has a chain of standard paper clips and Malia has a chain of jumbo paper clips
- If Malia told Sasha the length of an object in jumbo paper clips, how could Sasha change that number into standard clip lengths?

Partners

(slide 13) **Proportional Links**

Ask participants to find a partner. Locate the Proportional Links handout and read the scenario (Module Two, Handout Four).

Sasha has a chain of standard paper clips and Malia has a chain of jumbo paper clips. If Malia told Sasha the length of an object in jumbo paper clips, how could Sasha change that number into standard clip lengths?

- Ask participants how they might solve this problem.
- Give each pair of participants both types of paper clips.
- Encourage participants to build chains of paper clips.
- Have participants lay the chains side by side with ends matching to observe and record how long in standard paper clips the given amount of jumbo paper clips are.
- Have participants complete the handout.

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State Board of Education (Department of Public Instruction)

Proportional Links

- What patterns do you see in the table?
- What patterns do you see in the graph?
- How many times bigger is the standard paper clip number than the jumbo paper clip number?
- How can you use the patterns to determine the length of an object in standard paper clips if you know its length in jumbo paper clips?

Partners

(slide 14) **Proportional Links**

What patterns do you see in the table? In the graph?

- For every two jumbo paper clips there are three standard paper clips
- The points form a straight line
- On the line, you go up three and right two

How many times bigger is the standard paper clip number than the jumbo paper clip number?

1.5

How can you use the patterns to determine the length of an object in standard paper clips if you know its length in jumbo paper clips?

- Multiply the jumbo paper clip number by 1.5
- Multiply the jumbo paper clip number by three and then divide by two

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Proportional Links

- If Sasha measured an object with a length of 90 standard paper clips, how long will the object be if Malia measured it in jumbo paper clips?
- If Malia measured an object with a length of 55 jumbo paper clips, how long will the object be if Sasha measured it in standard paper clips?

Partners

(slide 15) **Proportional Links**

If Sasha measured an object with a length of 90 standard paper clips, how long will the object be if Malia measured it in jumbo paper clips?

60 jumbo paper clips

If Malia measured an object with a length of 55 jumbo paper clips, how long will the object be if Sasha measured it in standard paper clips?

82.5 standard paper clips

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Proportional Links

- How does this activity develop the concept of ratios as a rate of change?
- How does this activity develop the concept of conversion from one unit to another?

Partners

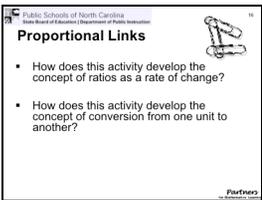
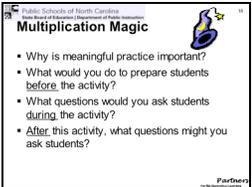
(slide 16) **Proportional Links**

How does this activity develop the concept of ratios as a rate of change?

- Rates are comparisons of different measures; here we are comparing jumbo paper clip lengths to standard paper clip lengths.
- For every 3 standard paper clips there are 2 jumbo paper clips
- For every one jumbo paper clip there are 1.5 standard paper clips
- Ask for someone to connect this rate to the graph - Graphically, you can see that you go up 3 (increase in standard) and right 2 (increase in jumbo)

How does this activity develop the concept of conversion from one unit to another?

- The conversion rate is found by using a ratio to compare the quantity in one system to an

	<p>equal value in another system (3 jumbo paper clips to 2 standard paper clips).</p>	
	<p>(slide 17) Multiplication Magic Give each group of four a set of <i>Multiplication Magic</i> cards (Module Two, Handout Five).</p> <ul style="list-style-type: none"> • Provide direction to participants on how to find the missing numbers. The first two numbers in each row or column are multiplied together to find the third number. • Using small sticky notes, participants can record their answers and place the sticky note in the appropriate space on the square. Participants may also record answers on the recording sheet. • Encourage participants to try more than one card. 	
	<p>(slide 18) Multiplication Magic <i>Why is meaningful practice important?</i></p> <ul style="list-style-type: none"> • Skill development • Mastery of a procedure or several procedures • Student engagement <p><i>What would you do to prepare students <u>before</u> this activity?</i></p> <ul style="list-style-type: none"> • Multiplying fractions • Inverse operations (undo multiplication, divide) • Answers will vary <p><i>What questions would you ask students <u>during</u> this activity?</i></p> <ul style="list-style-type: none"> • What strategy did you use to find the missing part? • Are there other ways that you might solve the problem? • Did you need to solve every problem to find a final solution? • Did you always use multiplication to find the missing numbers? <p><i><u>After</u> this activity, what questions might you ask?</i></p> <ul style="list-style-type: none"> • What strategies did you use to find the missing numbers? • What processes did you use to find the missing part and are the processes reasonable? • Was one square more challenging than another? Explain. 	
	<p>(slide 19) Music Match</p>	

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Music Match

- While the music is playing, walk around the room, exchanging cards with other participants
- When the music stops, find your group of three (equivalent fraction, decimal, percent)



Partners

Teachers will need to choose appropriate music for their class. Any song may be used.

- Give each participant a fraction, decimal, or percent card (Module Two, Handout Six). While the music is playing, participants will walk around the room exchanging fraction, decimal, and percent cards.
- When the music stops, participants are to find their groups. In each group of three should be an equivalent fraction, decimal, and percent.
- Play the music again, and have participants repeat the process.

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Music Match

- Do your students have difficulties with equivalence in different forms?
 - Is the difficulty recognizing equivalent values or creating equivalent values?
- Why is it important for students to know fraction, decimal, percent equivalence?



Partners

(slide 20) **Music Match**

Do you your students have difficulties with equivalence in different forms?

- In recognizing equivalent values?*
- In creating equivalent values?*

Answers will vary.

Why is it important for students to know fraction, decimal, and percent equivalence?

- Recognizing this equivalences help students estimate and operate with a variety of rational numbers.
- Students should be able to work flexibly with fractions, decimals, and percents.
- It is important for students to understand the relationships that exist between the different symbolization of rational numbers.

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Concept Clarification

- Percent is fundamentally different from a fraction or decimal
 - Percents are not considered values but operators
 - We should look at percent in the context of percent of some value
- Discuss how this fact should affect your instruction



Partners

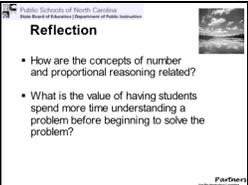
(slide 21) **Concept Clarification**

A percent is fundamentally different from a fraction or decimal. Percents are not considered values, but operators. We should look at percent in the context of “percent of some value”.

Discuss how this fact should affect your instruction.

Last summer, we placed percents on a number line; this provoked the conversation about the value of 25% and where it should be located on the number line. 25% can not be placed on a number line, but 25% of a given value can be. Placing 25% at the .25 mark on a number line is incorrect unless we recognize that we are equating 25% of 1 with the value .25.

This is a fine point since translating 25% into “25 parts out of 100” allows the substitution of .25 or 25/100 or 1/4 into an operation involving percents (25% of 10 replaced with .25x10). So we

	<p>seem to assume equality in order to be able to use a percent as an operator.</p>	
	<p>(slide 22) Reflection Have participants turn to their neighbor and respond to the two questions on the slide.</p> <p><i>How are the concepts of number and proportional reasoning related?</i></p> <p><i>What is the value of having students spend more time understanding a problem before beginning to solve the problem?</i></p>	
	<p>(slides 23-26) Credits for project and closing slides</p>	