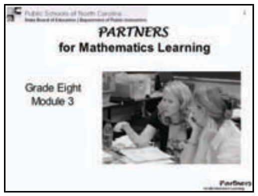
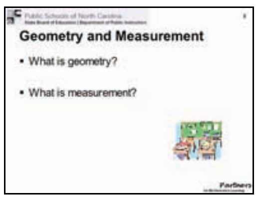
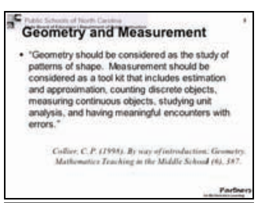
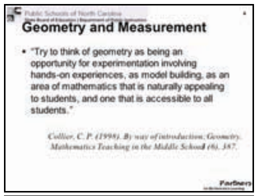
General Materials and Supplies: Handouts 10 and 11		8 ½ x 11 paper	white boards	markers
chalk	tape	tape measure	ruler	yard stick
meter stick	tennis balls in the container	shoe boxes	scissors	snap cubes
graphing calculators				

Slide	Tasks/Activity	Personal Notes
	(slide 1) **Grade Eight** Module 3 - introductory slide. Welcome back!	
	(slide 2) **Geometry and Measurement** *What is geometry?* *What is measurement?* Allow participants a few moments to describe geometry and measurement. These questions are used to get their minds on the topic, not to establish mathematically correct definitions. Some possible definitions of <u>geometry</u> include: The study of figures and their characteristics Relationships among 2-dimensional and 3-dimensional figures Some possible definitions of <u>measurement</u> include: Determining size and capacity Quantifying attributes to aid in describing an object – shape and size	
	(slide 3) **Geometry and Measurement** • *"Geometry should be considered as the study of patterns of shape. Measurement should be considered as a tool kit that includes estimation and approximation, counting discrete objects, measuring continuous objects, studying unit analysis, and having meaningful encounters with errors.* *Collier, C. P. (1998). By way of introduction: Geometry. Mathematics Teaching in the Middle School. 3 (6), 387.*	

Ask the participants the following questions:

- Is there any part of this quote that you especially agree with?
- Does this challenge your thinking about geometry and measurement?
- Is anything new to you?
- Do you have questions about what is on the slide?



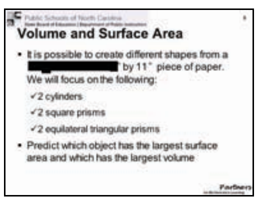
(slide 4) **Geometry and Measurement**

- *"Try to think of geometry as being an opportunity for experimentation involving hands-on experiences, as model building, as an area of mathematics that is naturally appealing to students, and one that is accessible to all students."*

Collier, C. P. (1998). By way of introduction: Geometry. Mathematics Teaching in the Middle School. 3 (6), 387.

Ask the participants the following questions:

- Is there any part of this quote that you especially agree with?
- Does this challenge your thinking about geometry and measurement?
- Is anything new to you?
- Do you have questions about what is on the slide?



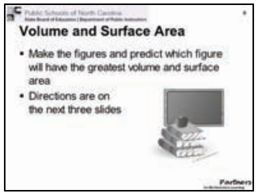
(slide 5) **Volume and Surface Area**

MATERIALS: 6 pieces of 8 ½" x 11" paper per group, tape, white board and markers or more paper for recording measurements

- *It is possible to create different shapes from a single standard 8 ½" by 11" piece of paper. We will focus on the following:*
  - ✓ *2 cylinders*
  - ✓ *2 square prisms*
  - ✓ *2 equilateral triangular prisms*
- *Predict which object has the largest surface area and which has the largest volume.*

Last year we compared the volume of two cylinders formed from the same size sheet of paper. One cylinder was tall and thin, the other short and wide. Do you remember which one had the largest volume?

Estimate which of the six will have the greatest surface area and which will have the greatest volume. Though the figures do not have bases taped to them, include the bases when estimating surface area.

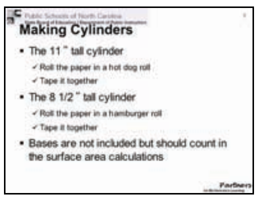


(slide 6) **Volume and Surface Area**

- *Make the figures and predict which figure will have the greatest volume and surface area.*
- *Directions are on the next three slides*

As participants compare figures of each type they may be able to predict that each of the shorter figures of the two will have the larger surface area, since the bases are larger while the lateral surface areas are equal (as they come from the same sized piece of paper.

Instruct each group to make the six objects and calculate volume and surface area. Instructions for each object are on the following slides.



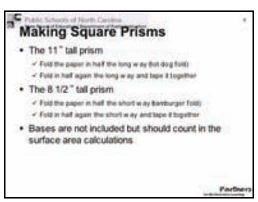
(slide 7) **Making Cylinders**

To Make the Cylinders: (the bases are not included in the construction)

The 11" tall cylinder-
Roll the paper in a "hot dog" roll.
Tape it together.

The 8 1/2" tall cylinder –
Roll the paper in a "hamburger" roll.
Tape it together.

The bases are not included in the figure, but should count in the surface area calculation.



(slide 8) **Making Square Prisms**

To Create the Square Prisms: (the bases are not included in the construction)

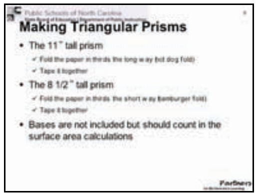
The 11" tall prism-
Fold the paper in half the long way (hot dog fold).
Fold in half again the long way.
Tape it together.

The 8 ½ " tall prism-
Fold the paper in half the short way (hamburger fold).
Fold in half again the short way.

Tape it together.

The bases are not included in the figure, but should count in the surface area calculation.

Note: once the folds are made the creases mark the fold lines that should be used to make the sides of the prism. Participants will have to fold along a line in the opposite direction to create a closed figure.

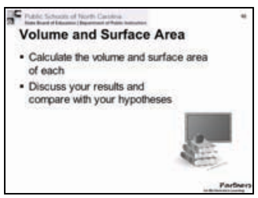


(slide 9) **Making Triangular Prisms**

To Create the Equilateral Triangular Prisms: (the bases are not included in the construction)

The 11" tall prism-
Fold the paper in thirds the long way (hot dog fold).
Tape it together.

The 8 ½ " tall prism-
Fold the paper in thirds the short way (hamburger fold).
Tape it together.
The bases are not included in the figure, but should count in the surface area calculation.



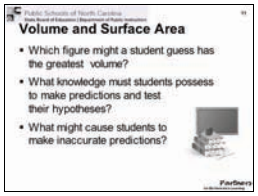
(slide 10) **Volume and Surface Area**

- *Calculate the volume and surface area of each.*
- *Discuss your results and compare with your hypotheses.*

When most groups are finished, have people put answers on a chart, either on their own paper or a white board.
Discuss their answers as a group.

Approximate Volume and Surface Area

	Volume	Surface Area
Tall cylinder	63.28 in^3	105.04 in^2
Short cylinder	81.89 in^3	112.77 in^2
Tall square prism	49.67 in^3	102.53 in^2
Short square prism	64.28 in^3	108.63 in^2
Tall triangular prism	38.24 in^3	100.45 in^2
Short triangular prism	49.48 in^3	105.14 in^2



(slide 11) **Volume and Surface Area**

- *Which figure might a student guess has the greatest volume?*

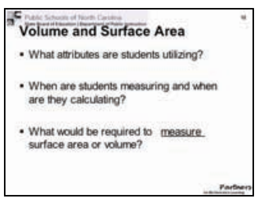
  Students may guess that the tall square prism contains the greatest volume.
  If students do not consider the bases when calculating surface area, they may say that all of the figures have the same surface area, which in that case, is correct.

- *What knowledge must students possess to make predictions and test their hypotheses?*

  Measurement
  Volume, area and surface area
  Classifications of geometric solids
  Properties of geometric solids

- *What might cause students to make inaccurate predictions?*

  Failure to reason correctly about the same size paper forming the sides of all figures
  Assuming "taller" means more



(slide 12) **Volume and Surface Area**

- *What attributes are students utilizing?*

  Length, area, and volume

- *When are students measuring and when are they calculating?*

  Volume and area – they are calculating the measures, but not actually measuring those attributes

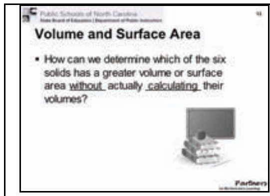
- *What would be required to measure surface area or volume?*

  Using square units such as square tiles for surface area and cubic units such as cubic centimeter blocks for volume

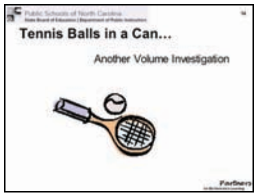
(slide 13) **Volume and Surface Area**

How can we determine which of the six solids has a greater volume or surface area without calculating the measures?

If participants want to compare the area of each base symbolically, refer to Handout #10.

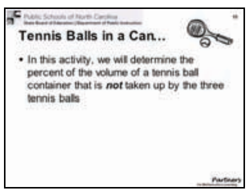


Participants may also suggest ideas like the following:
Pour something in the container to see which has the greater volume.
Trace the net onto grid paper and compare the surface areas.
When thinking of surface area, only the bases make the difference.



(slide 14) **Tennis Balls in a Can…** *Another Volume Investigation*
MATERIALS: one can of tennis balls per group, tape measure, ruler

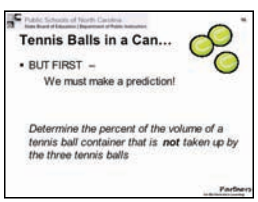
At this time, pass out one can of tennis balls to each table.
Ask participants to consider the amount of space the tennis balls occupy and the amount of space in the can, not occupied by the tennis balls.



(slide 15) **Tennis Balls in a Can…**

- *In this activity, we will determine the percent of the volume of a tennis ball container that is **not** taken up by the three tennis balls.*

Move to the next slide quickly before the participants start their calculation.



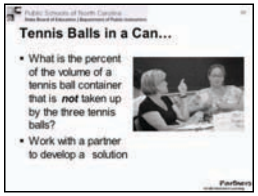
(slide 16) **Tennis Balls in a Can…**

- *BUT FIRST –*

  We must make a prediction!

  *Determine the percent of the volume of a tennis ball container that is **not** taken up by the three tennis balls.*

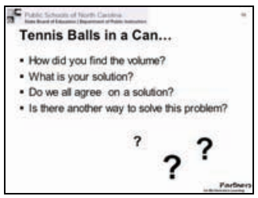
Allow enough time for each participant to independently make a prediction.



(slide 17) **Tennis Balls in a Can…**

- *What is the percent of the volume of a tennis ball container that is **not** taken up by the three tennis balls?*
- *Work with a partner to develop a solution*

Participants should work in pairs to answer this question. Ask participants to be prepared to share their ideas with the group. Circulate through the group and try to identify at least two different solution strategies being used by participants.



(slide 18) **Tennis Balls in a Can…**

- *How did you find the volume?*
- *What is your solution?*
- *Do we all agree on a solution?*
- *Is there another way to solve this problem?*

Ask a few pairs to share their solutions, calling on pairs with differing solution methods. After a sufficient amount of time, discuss the solutions.

This problem can be solved analytically as follows:

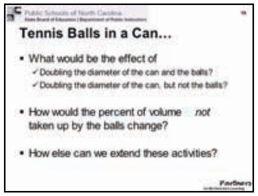
Volume of the can = $\pi r^2 h = \pi r^2(3d) = \pi r^2(6r) = 6\pi r^3$, where r represents the radius of the can/balls, d represents the diameter of the balls/can, h represents the height of the can.

Volume of the balls = $3 \cdot \left(\frac{4}{3}\right)\pi r^3 = 4\pi r^3$**,** where r represents the radius of the balls/can.

(Remember volume of a sphere is $V = \left(\frac{4}{3}\right)\pi r^3$.)

(Volume of the can) – (Volume of the balls) = $6\pi r^3 - 4\pi r^3 = 2\pi r^3$

% of volume not taken up by the balls = 33%



(slide 19) **Tennis Balls in a Can…**

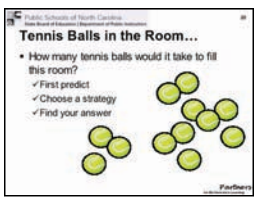
What would be the effect of

Doubling the diameter of the can and the balls?
The percent would remain the same.

Doubling the diameter and can, but not the balls?
The percent would change to 83%

How would the % of the volume not taken up by the balls change?

How else can we extend this activity?
Suggest considering 12 tennis balls instead of just 3. Is there any difference volume-wise with the packaging between laying the 12 tennis balls in a box versus laying the 12 in a super-tall can? You may spend a short amount of time discussing the above problem, but it is not intended that you spend time solving it.



(slide 20) **Tennis Balls in the Room…**

MATERIALS**:** Shoe boxes, tennis balls, meter stick/yard stick

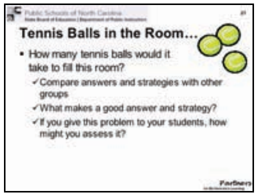
- *How many tennis balls would it take to fill this room?*
  - ✓ *First Predict*
  - ✓ *Choose a strategy*
  - ✓ *Find your answer*

Encourage groups to work together to make an estimate, create a strategy, and find an answer. It is reasonable to assume that the tennis balls will always be "stacked" as they are in a can of tennis balls.

Hand out shoe boxes and a couple of tennis balls to help participants find their answer. Breaking it down into a smaller problem can be a strategy to help them solve the larger problem. When doing this with students, it may be important to talk about how the shoe boxes can help.

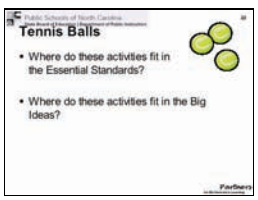
Note: This activity provides only an estimate of the number of tennis balls that can fill the room.

Those who are interested in finding a more exact answer can google "sphere packing" for more information.



(slide 21) **Tennis Balls in the Room…**

- *How many tennis balls would it take to fill this room?*
  - ✓ *Compare answers and strategies with other groups.*
    Answers will vary depending on the size of the room.
  - ✓ *What makes a good answer and strategy?*
    Some will say that a good answer is mathematically correct.
    Some will say that a good answer communicates problem solving skills, though there may be minor errors in the computation.
    Good strategies might include breaking the room into smaller sections or thinking through the process as a multi-step procedure.
  - ✓ *If you give this problem to your students, how might you assess it?*
    Suggestions may include use a rubric



(slide 22) **Tennis Balls**

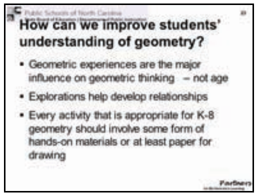
Where does this fit in the Essential Standards?

Where does this fit in the Big Ideas?

Teachers may state any of the following.

- The attribute to be measured determines the unit and the tool
- Measurements are estimates; the more precise the tools/units, the closer one can get to the actual measure
- Measurements are accurate to the extent that the appropriate units/tools are used properly
- Perimeter/circumference and area of 2-D figures are related to surface area and volume of 3-D figures
- Formulas that describe attributes of geometric figures can be utilized for indirect measurements and problem solving

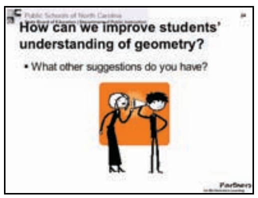
Remember, this activity is to get teachers talking and thinking about the Essential Standards and the Big Ideas and how the mathematics relates. There is no right or wrong answer.



(slides 23) **How can we improve students' understanding of geometry?**
Ask a participant to read the bullets on the slide. In small groups have participants discuss each idea- do they agree or disagree, and why?

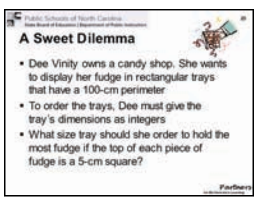
- *Geometric experiences are the major influence on geometric thinking – not age.*
- *Explorations help develop relationships.*
- *Every activity that is appropriate for K-8 geometry should involve some form of hands-on materials or at least paper for drawing.*

Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc.



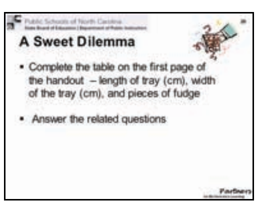
(slide 24) **How can we improve students' understanding of geometry?**
What other suggestions do you have?

Ask participants to share their ideas. Responses will vary.



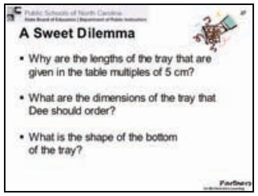
(slide 25) **A Sweet Dilemma**
MATERIALS: Handout #11 (Sweet Dilemma), ruler, scissors, 8 ½ x 11 paper, snap cubes (optional)
Dee Vinity owns a candy shop. She wants to display her fudge in rectangular trays that have a 100-cm perimeter. To order the trays, Dee must give the tray's dimensions as integers. What size tray should she order to hold the most fudge if the top of each piece of fudge is a 5-cm square?

Have a participant read the problem as materials are handed out.



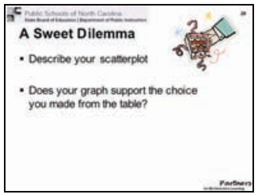
(slide 26) **A Sweet Dilemma**
Complete the table on the first page of the handout – length of tray (cm), width of tray (cm), and pieces of fudge. Then answer the related questions.

Have participants work together at their tables. Allow enough time for participants to complete their scatterplots.



(slide 27) **A Sweet Dilemma**
Why are the lengths of the tray that are given in the table multiples of 5 cm?
What are the dimensions of the tray that Dee should order?
What is the shape of the bottom of the tray?

Ask for responses to these questions from different groups.

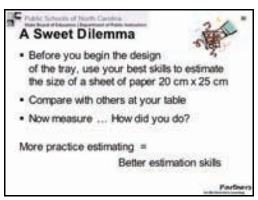


(slide 28) **A Sweet Dilemma**
Describe your scatterplot.
Does your graph support the choice you made from the table?

Allow participants to share responses across groups. Then ask for a volunteer to share a scatterplot and table, explaining the relationships between the two.

(slide 29) **A Sweet Dilemma – A New Tray**
Dee would also like to package her fudge to sell in solid blocks, but she is not sure what size container would be most efficient for the blocks. She wants you to design an open tray (which will be covered by cellophane) that she can use to package her block fudge.
The material for the tray is a 25 cm x 20 cm rectangle.

Move to the next slide for instructions.

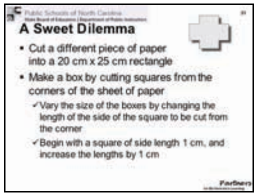


(slide 30) **A Sweet Dilemma**

- *Before you begin the design of the tray, use your best skills to estimate the size of a sheet of paper 20 cm x 25 cm.*
- *Compare with others at your table.*
- *Now measure… How did you do?*

More practice estimating =
Better estimation skills

This slide and the next slide are not on the worksheet. Keep the whole group focused on these questions before moving on to the worksheet.

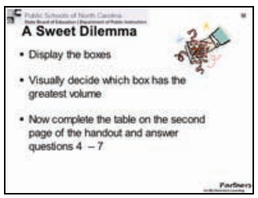


(slide 31) **A Sweet Dilemma**

- *Cut a different piece of paper into a 20 cm x 25 cm rectangle.*
- *Make a box by cutting squares from the corners of the sheet of paper.*
  - ✓ *Vary the size of the boxes by changing the length of the side of the square to be cut from the corner.*
  - ✓ *Begin with a square of side length 1 cm, and increase the lengths by 1 cm.*

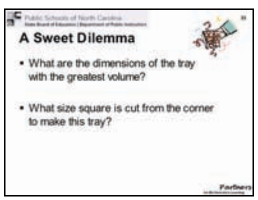
Note: This could be done with snap cubes which will be helpful making the connection between the dimensions and the generalized expressions and equations in #8 and #9 in the handout.

What are the dimensions of the largest square that can be cut from the rectangle?
Answer: A square of side length 9 cm.



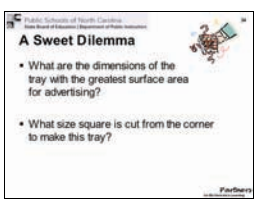
(slide 32) **A Sweet Dilemma**

- *Display the boxes.*
- *Visually decide which box has the greatest volume.*
- *Now complete the table on the second page of the handout and answer questions 4 – 7.*



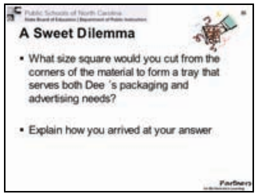
(slide 33) **A Sweet Dilemma**

- *What are the dimensions of the tray with the greatest volume?*
- *What size square is cut from the corner to make this tray?*



(slide 34) **A Sweet Dilemma**

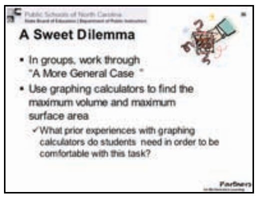
- *What are the dimensions of the tray with the greatest surface area for advertising?*
- *What size square is cut from the corner to make this tray?*



(slide 35) **A Sweet Dilemma**

- *What size square would you cut from the corners of the material to form a tray that serves both Dee's packaging and advertising needs?*
- *Explain how you arrived at your answer.*

Answers should include justification for how the container satisfies both packaging and advertising needs.

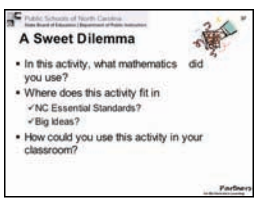


(slide 36) **A Sweet Dilemma**

- *In groups, work through "A More General Case."*
- *Use graphing calculators to find the maximum volume and maximum surface area*
- *What prior experiences with graphing calculators do students need in order to be comfortable with this task?*

Point out the relationship between the nets and the expressions.

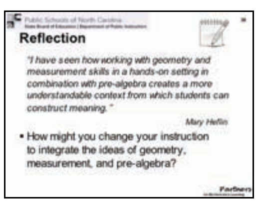
(If LEA's are willing and participants are interested, finding the maximum volume using calculus and derivatives would be a good discussion. That is what this activity leads to in the upper grades.)



(slide 37) **A Sweet Dilemma**

- *In this activity, what mathematics did you use?*
- *Where does this activity fit in*
  - ✓ *Essential Standards?*
  - ✓ *Big Ideas?*
- *How could you use this activity in your classroom?*

Answers will vary.



(slide 38) **Reflection**

"I have seen how working with geometry and measurement skills in a hands-on setting in combination with pre-algebra creates a more understandable context from which students can construct meaning."
Mary Heflin

Collier, C. P. (1998). By way of introduction: Geometry. *Mathematics Teaching in the Middle*

	School. 3 (6), 415. *How might you change your instruction to integrate the ideas of geometry, measurement, and pre-algebra?* Have participants talk with each other at their tables. Then ask for one participant at each table to share with the whole group a summary of what their table group had said.	
	(slide 39-42**) Credits and closing slides**	