Volume and Surface Area

Can we determine which of the six solids has a greater volume without actually calculating their volumes?

The area of the bases makes the difference.

Calculate the area of each base region symbolically and compare.

Consider a circle: *l* is the circumference of the circle.
$l = 2\pi r$
To find the area, use the formula $A_c = \pi r^2$

But first, what is *r*?
$r = \frac{l}{2\pi}$

Substitute for *r*.
$A_c = \pi\left(\frac{l}{2\pi}\right)^2$

Simplify.
$A_c = \frac{l^2}{4\pi}$

Consider a square: *l* is the perimeter of the square.
$l = 4s$

What is *s*?
$s = \frac{l}{4}$

What is the area of the square if $A_s = s^2$?
$A_s = (\frac{l}{4})^2$
$A_s = \frac{l}{16}^2$

Consider an equilateral triangle: *l* is the perimeter of the triangle.
$l = 3s$

What is *s*?
$s = \frac{l}{3}$

What is the area of an equilateral triangle?

$$A_T = \frac{1}{2}\left(\frac{l}{3} \cdot \frac{l\sqrt{3}}{6}\right)$$

$$A_T = \frac{l^2\sqrt{3}}{36}$$

Now compare the different areas

$A_c = \frac{l^2}{4\pi}$ $A_s = \frac{l}{16}^2$ $A_T = \frac{l^2\sqrt{3}}{36}$

Divide each area by l^2.

$$\frac{1}{4\pi} > \frac{1}{16} > \frac{\sqrt{3}}{36}$$

Given equal circumference (perimeter) and height, the volume of a cylinder will always be greater than the volume of the square or triangular prism.