Next Steps and Instructional Moves

The intended purpose of this document is to provide teachers with a tool to determine student understanding and suggest instructional moves that may help guide a student forward in their learning of a concept or standard. This guide is not an exhaustive list of strategies.

<table>
<tr><td>Third Grade: Cluster 4
Making Sense of Multiplication and Division</td></tr>
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NC.3.OA.1 For all products of whole numbers with two factors up to and including 10:
- Interpret the factors as representing the number of equal groups and the number of objects in each group.
- Illustrate and explain strategies including arrays, repeated addition, decomposing a factor, and applying the commutative and associative properties.

NC.3.OA.2 For whole-number quotients of whole numbers with a one-digit divisor and a one-digit quotient:
- Interpret the divisor and quotient in a division equation as representing the number of equal groups and the number of objects in each group.
- Illustrate and explain strategies including arrays, repeated addition or subtraction, and decomposing a factor.

NC.3.OA.3 Represent, interpret, and solve one-step problems involving multiplication and division.
- Solve multiplication word problems with factors up to and including 10. Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem.
- Solve division word problems with a divisor and quotient up to and including 10. Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem.

NC.3.OA.6 Solve unknown-factor problem by using division strategies and/or changing it to a multiplication problem.

NC.3.OA.7 Demonstrate fluency with multiplication and division with factors, quotients and divisors up to and including 10.
- Know from memory all products with factors up to and including 10.
- Illustrate and explain using the relationship between multiplication and division.
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

NC.3.OA.8 Solve two-step word problems using addition, subtraction, and multiplication, representing problems using equations with a symbol for the unknown number.

NC.3.OA.9 Interpret patterns of multiplication on a hundreds board and/or multiplication table.

NC.3.NBT.3 Use concrete and pictorial models, based on place value and the properties of operations, to find the product of a one-digit whole number by a multiple of 10 in the range 10-90.

Next Steps and Instructional Moves

| Not Yet	Students that are consistently scoring "Not Yet" on multiplication and division tasks could have a variety of errors. Students may have a difficult time identifying and/or using an effective strategy to multiply and/or divide. Students may not have any concept of multiplication and/or division and how they relate to addition/subtraction. Students may demonstrate barriers related to fluency with multiplication and/or division.

Next Steps:

For students having difficulty finding the product of factors:

- Pose tasks where students need to build equal groups with counters or cubes.
- Discuss multiplication equations with students and the idea that the two factors represent different quantities- one factor is "the number of groups" and the other factor is "the number in each group"

3 x 5 = 15
3 "groups of" 5
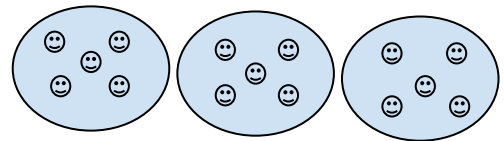
- Use smaller numbers to allow students opportunities to easily model tasks. Ex: tasks involving groups of 2, 5, or 10 **OR** 2 groups, 5 groups or 10 groups of objects.

- Pose tasks where students use tiles to create rows and columns to build arrays, then draw the arrays on grid paper (reinforce that the number of rows because a factor, and the number in each row becomes the other factor in an equation). While building arrays, ask students to write repeated addition equations to relate repeated addition back to multiplication.
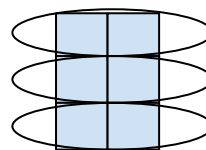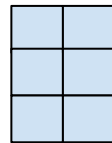
3 rows with 2 tiles in each row

3
3
3
3
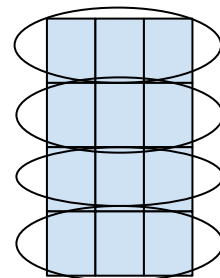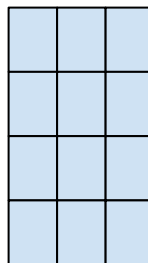
3 + 3 + 3 + 3

- Pose tasks involving equal groups where students can circle the addends in repeated addition equations. Help reinforce that the number of circles becomes the "number of groups" factor, and the number in each circle becomes the "number in each group" factor of multiplication

2 + 2 + 2 + 2 + 2 = 1

5 groups of 2 = 5 x 2 = 10

- Pose daily number talks and routines focused on multiplication and skip counting
- Discuss the commutative and associative property of addition and relate it to those same properties of multiplication

Next Steps and Instructional Moves

Not Yet (continued)	

Commutative Property
Order does not change the total

2 + 6 = 8	2 x 6 = 12
6 + 2 = 8	6 x 2 = 12

Associative Property
Grouping does not change the total

| (2 + 3) + 4	2 + (3 + 4)
5 + 4	2 + 7
9	9

For students having difficulty understanding division:
- Give students a quantity of tiles (e.g., 12, 16, or 20) and a specific number of groups to make and have them make equal groups of tiles.
- Give students square tiles (e.g., 12 or 16) and pose tasks where they must make various sizes of arrays. Ex: You have 16 chairs. What are the different ways you can put the chairs into equal sized groups? Find as many possible combinations as you can.
- Pose division word problems and discuss with students that the quotient is either the number of equal groups created or the number of objects in 1 of the equal groups.
- Adjust numbers used to match students' needs (start with 2's, 5's, 10's)
- Pose division tasks and encourage students to consider using repeated subtraction. Ex: You have 42 pencils. If you put 7 in each bag how many bags of pencils will you have?
- Pose daily number talks and routines focused on division and its relationship to multiplication and/or subtraction
- Solving division problems [lesson](#)

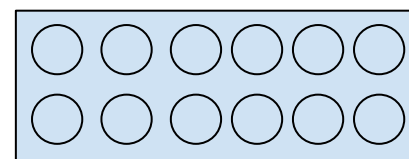**For students having difficulty interpreting multiplication and/or division word problems:**
- Pose tasks and have students retell the problem in their own words. Ask students to identify what is known in the task and what the question is asking.
- Pose tasks and encourage students to use either manipulative (tiles, counters, plates) or pictures/arrays to represent the tasks.
- Discuss with students the actions associated with multiplication and division: Multiplication focuses on determining a total given a group size and the number of equal groups. Division focuses on determining either the group size or the number of equal groups when given one of those quantities as well as the total number of objects.
- Solve problems and encourage the use of strategies such as the bar model (see picture) and/or part-part-whole mats.
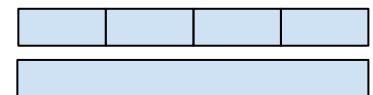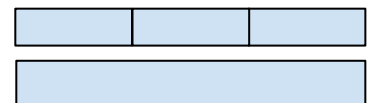
For students having difficulty solving for unknown factor:
- Use plates and counters to have students determine missing factor
- Help them understand commutative property to rearrange factors to create equal groups easier (might be easier to draw the number of groups first)
  ___ x 5 =15 (how many groups of 5 will make 15?); 5 x __= 15 (create 5 groups, fair-share 15

2 rows of 6 circles is 12 circles
2 x 6 = 12
12 circles divided into 2 rows is 6 circles in each row

Next Steps and Instructional Moves

Not Yet (continued)	counters to figure out how many will be in each group)

For students having difficulty with multiplication and/or division fluency:
- Pose related tasks with the same numbers in the context of arrays. For example:
  - Mr. Mauk had a football card collection. He displayed the cards in 4 rows with 5 cards in each row. How many football cards are in his collection?
  - Mr. Mauk had 20 football cards in his collection. He displayed them into 4 rows. How many cards are in each row?
- Present students with arrays and have students write an equation that matches and determine the total number of objects. Give students an equation and have students build an array and determine the total number of objects.
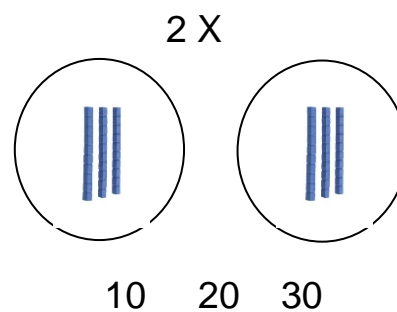- Skip counting lesson

For students having difficulty solving two-step word problems:
- Allow students to use counters to "act out" what is happening in the story. Read the whole problem, them reread the problem, stopping at each action. Have students create an equation based on the action and discuss what each number of the equation represents from the story. Continue until all steps are finished.
- Pose a word problem and have students retell the problem to a classmate or the entire class. Have students identify what the problem is asking and whether they will need 1 or 2 steps to determine the answer.
- Provide exposure to a variety of problems that used combinations of all 4 operations
- Give students a set of one-step and two-step word problems and have them sort the problems into one-step or two-step word problems, justifying their sort.

For students having difficulty interpreting patterns of multiplication:
- Use counters to cover or allow students to highlight multiples of a factor on a hundreds board and/or a multiplication table.
- Have students make lists of multiples by using repeated addition.
- Square table tops lesson

2 X

10   20   30

For students having difficulty multiplying with multiples of 10's:
- Have students practice skip counting by 10's first, writing down all the multiples of 10 they say.
- Have students discuss connection between counting by 2's and 20's (how can knowing x2 facts help with x20?)
- Adjust numbers based on evidence of students' understanding
- Allow students to use base-ten blocks and plates to model equation

Next Steps and Instructional Moves

Progressing	Students that are consistently scoring "Progressing" have a strategy to use for multiplication and division; however, they have not mastered this strategy or may also still struggle with the difference between multiplication and division. Students may also be fluent with some multiplication/division facts, however still need to demonstrate mastery. Students may also be able to solve one-step word problems, but have not mastered two-step word problems (or fail to solve for the second step).

Next Steps:

For students having difficulty finding the product of more challenging factors:
- Pose tasks and encourage students to use manipulatives to model tasks with tiles, counters, or cubes. Have students write equations that match the representation and the task.
- Pose tasks and encourage students to draw arrays that match the task. Discuss with students the strategy of breaking an array apart and determining the size of the smaller arrays in order to find the size of the larger array.. E.g., 8x7 could be broken into an 8x5 array and an 8x2 array to determine that 8x7 = 8x5 + 8x2. .

For students having difficulty understanding division:
- Give students a quantity of tiles and a specific number of groups and have them divide equally (fair share) into groups to find the quotient; create equation based on representation
- Give students square tiles to divide into equal rows and columns (relate to arrays) and/or use graph paper; create equation based on representation
- Help students understand the quotient is either the number of equal groups or the number in each group; or the number of rows or the number in each row
- Pose tasks that involve the action of repeated subtraction. E.g., There are 20 birds in the park and they fly away in groups of 4. How many groups of birds flew away?
- Solving division problems [lesson](#)

For students having difficulty interpreting multiplication and/or division word problems:
- Pose tasks and have students retell the problem in their own words. Ask students to identify what is known in the task and what the question is asking.
- Pose tasks and encourage students to use either manipulative (tiles, counters, plates) or pictures/arrays to represent the tasks.
- Discuss with students the actions associated with multiplication and division: Multiplication focuses on determining a total given a group size and the number of equal groups. Division focuses on determining either the group size or the number of equal groups when given one of those quantities as well as the total number of objects.

For students having difficulty solving for unknown factor:
- Use plates and counters to have students determine missing factor
- Pose tasks related to the commutative property and discuss with students how they can use the relationship between multiplication and division. Ex: ___x 5 =15 (how many groups of 5 will make 15?); 5 x __= 15 (create 5 groups, fair-share 15 counters to figure out how many is in each

Next Steps and Instructional Moves

Progressing (continued)	group)

For students having difficulty with multiplication and/or division fluency:
- Product Compare Game: Students play with a partner. Each student pulls 2 number cards and finds the product. The student with the highest product wins a point. Keep playing the game for a set amount of time OR until a student has earned 10 points.
- Encourage students to build arrays for multiplication combinations that they have trouble recalling. Cut the arrays out so on one side it has the array and the dimensions and on the other sides students can have both equations written, such as 4x3=12 and 3x4=12.

6, 42, 7

6 x 7 = 42
7 x 6 = 42

For students having difficulty solving two-step word problems:
- Allow students to use counters to "act out" what is really happening in the story. Read the whole problem, them reread the problem, stopping at each action. Have students create an equation based on the action and discuss what each number of the equation represents from the story. Continue until all steps are finished.
- Pose a word problem and have students retell the problem to a classmate or the entire class. Have students identify what the problem is asking and whether or not they will need 1 or 2 steps to determine the answer.
- Provide exposure to a variety of problems that used combinations of all 4 operations
- Give students a set of one-step and two-step word problems and have them sort the problems into one-step or two-step word problems, justifying their sort.
- Solving two-step problems [lesson](#)

For students having difficulty interpreting patterns of multiplication:
- Use counters to cover or allow students to highlight multiples of a given factor on a hundreds board and/or a multiplication table, as they practice "counting by" that factor
- Discuss with students the relationship between shared multiples (for example: commonalities between multiples of 2 and multiples of 4).
- Square table tops [lesson](#)

For students having difficulty multiplying with multiples of 10's:
- Have students discuss connection between counting by 2's and 20's (how can knowing x2 facts help with x20?)
- Pose tasks and allow students to use tiles, cubes, or base ten blocks to model situations. Have students write equations and discuss the relationship between equations, representations, and the tasks.

Next Steps and Instructional Moves

Meets Expectation	Students that are consistently scoring "Meets Expectation" on demonstrate a consistent and fluent understanding of multiplication and division in their strategies and facts. Students can solve one and two-step word problems efficiently and effectively. Students can extend their understanding to larger numbers.
	Next Steps: ● Ask students to explore multi-digit multiplication and division starting with multiplying a teen number by a one-digit number AND dividing a two-digit number by a one-digit number where the quotient is a teen number. ● Allow students to apply properties in more complex computations, including distributive property while multiplying a two-digit number by a one-digit number. $$13 \times 6 =$$ $$(10+3) \times 6 =$$ $$(10 \times 6) + (3 \times 6) =$$ $$60 \ + \ 18 \ \ \ = 78$$ ● Give students either equations or representations, and have students create multiplication and division word problems that match.