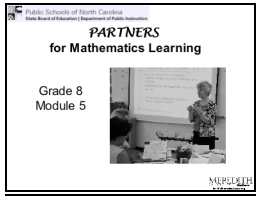
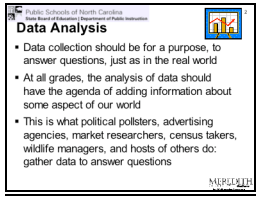
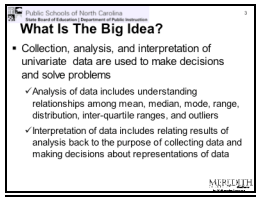
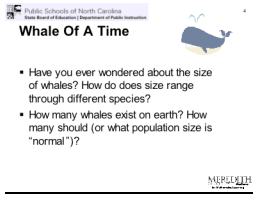
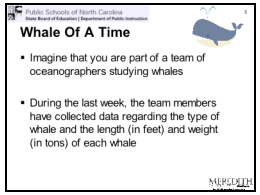
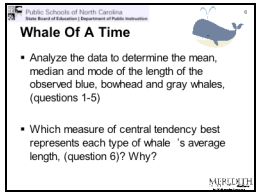
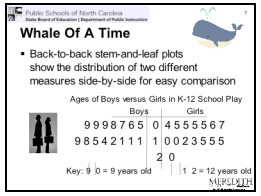
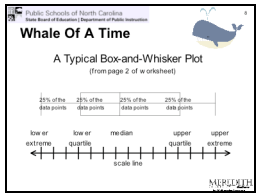
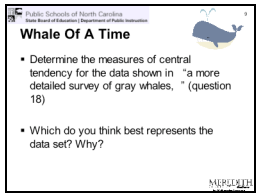
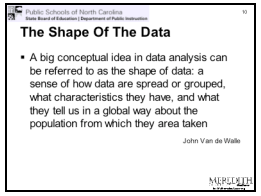
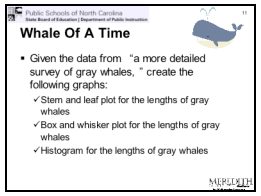
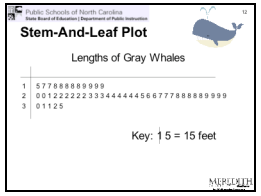
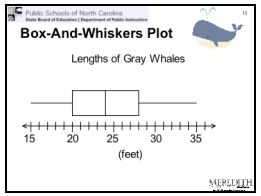
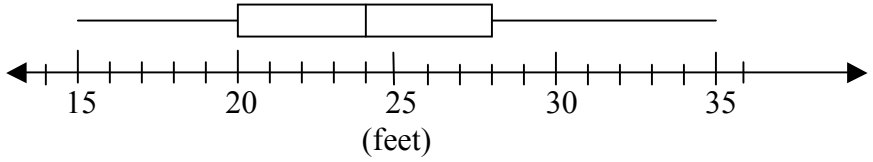
General Materials and Supplies: Whale of a Time Handout, Crow and the Pitcher Handout, Calculators, Paper, Graph paper, Navy Beans, Bag to hold beans, Permanent Markers, Beakers, Marbles, Water, Rulers, Chart Paper, Markers

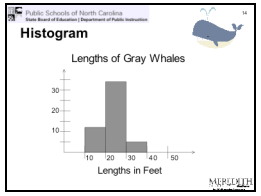
Slide	Tasks/Activity	Personal Notes
	(slide 1) **Grade Eight Module 5** Welcome back	
	(slide 2) **Data Analysis** *Data collection should be for a purpose, to answer a question, just as in the real world. At all grades, the analysis of data should have the agenda of adding information about some aspect of our world. This is what political pollsters, advertising agencies, market researchers, census takers, wildlife managers, and hosts of other do: gather data to answer to questions.* Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc. Spend just a minute or so discussing this quote. Ask participants what has been the purpose of data analysis within their own classrooms.	
	(slide 3) **What's the Big Idea?** Ask for a volunteer to read the points on the slide. *Collection, analysis, and interpretation of univariate data are used to make decisions and solve problems* • *Analysis of data includes understanding relationships among mean, median, mode, range, distribution, inter-quartile ranges, and outliers.* • *Interpretation of data includes relating results of analysis back to the purpose of collecting data and making decisions about representations of data.* The first part of our next activity will focus on utilizing univariate data to make decisions and	

	solve problems.	
	(slide 4) **Whale of a Time** *Have you ever wondered about the size of whales? How does size range through different species?* *How many whales exist on earth? How many should (or what population size is "normal")?* Ask what other questions might they have about whales? Ask participants if there is another species that they were interested in or some other animal their students are curious about.	
	(slide 5) **Whale of a Time** MATERIALS: Whale of a Time Handout, Calculators, Paper, Graph paper (optional) *Imagine that you are part of a team of oceanographers studying whales. During the last week, the team members have collected data regarding the type of whale and the length (in feet) and weight (in tons) of each whale.* Univariate data analysis is used to explore each variable in a data set separately. One can determine the range of data values as well as the measures of center of those values. <u>Note:</u> Use the slides and these notes to guide you as you complete the worksheet. There will be times when the activity on the slide is not on the worksheet. The worksheet will not be completed in the order in which it is written, use the slides as your guide.	
	(slide 6) **Whale of a Time** *Analyze the data to determine the mean, median and mode of the length of the observed blue, bowhead, and gray whales, (questions 1-5).* Group participants according to the type of calculator they are using, and then instruct them to utilize technology to find the answers. *Which measure of central tendency best represents each type of whale's average length, (question 6)? Why?* Have participants discuss and decide their response within their small groups before sharing	

	their ideas with the whole group. The measure of central tendency that best represents blue whale's length is the median. It is less influenced by the relatively small length of 14.9 and is most like the majority of the data. The measure that best represents the bowhead whale could be either the mean or the median because the data has no outliers. The measure that best represents the gray whale could be the mean, the median, or the mode because all are within the span of most of the data. Ask participants when a single average may not always be the best representative of a data set. For example, data could be such that 2 average values might be more appropriate (adolescent whales are typically __ ft., while adult whales are typically __ft. or maybe there are different averages for the sexes). Data that seems to be clustered in two groups (around two different means) is referred to as "bimodal".	
	(slide 7) **Whale of a Time** *Back-to-back stem-and-leaf plots show the distribution of two different measures side by side.* Ages of Boys versus Girls in K-12 School Play Boys | Girls 9 9 9 8 7 6 5 | 0 | 4 5 5 5 5 6 7 9 8 5 4 2 1 1 1 | 1 | 0 0 2 3 5 5 5 | 2 | 0 Key: 9 | 0 = 9 years old 1 | 2 = 12 years old Use this slide as a reference for questions 7 and 8. Ask a participant to explain the data represented by the graphs. Be sure everyone understands the representation before moving on.	
	(slide 8) **Whale of a Time** This slide contains "A Typical Box-and-Whisker Plot" from page 2 of the handout. Use this slide to discuss/ review the different parts of box-and-whiskers plots for the teachers. Most teachers are familiar with the parts of the plot as well as how to find them. Find the appropriate data points for questions 9-12. Question 13 asks participants to create two box-and-whiskers plot on one scale, being sure to label each one. One plot should be raised above the other so the two plots do not overlap. Participants should complete #13-14. Then discuss their graphs.	

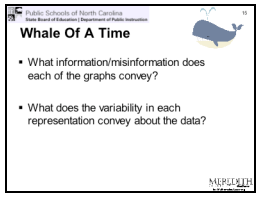
	Say, "Since we are focused on univariate data right now, the next questions will not be discussed until later in this module. Please skip to question 18."	
	(slide 9) **Whale of a Time** *Determine the measures of central tendency for the data shown in "a more detailed survey of gray whales," (question 18).* *Which do you think best represents the data set? Why?* Discuss answers to question 18.	
	(slide 10) **The Shape of the Data** *A big conceptual idea in data analysis can be referred to as the shape of data: a sense of how data are spread or grouped, what characteristics they have, and what they tell us in a global way about the population from which they are taken.* Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc. Students should learn that different graphs provide different snapshots of the data. For the particular question being answered, the choice of graphs is made around the notion of the shape of the data. What do each type of graph show about shape?	
	(slide 11) **Whale of a Time** *Given the data from "a more detailed survey of gray whales", groups of participants should create the following graphs:* 1. *Stem-and-leaf plot for the lengths of gray whales.* 1. *Box-and-whisker plot for the lengths of gray whales.* 2. *Histogram for the lengths of gray whales.* Stem-and-Leaf plots, Box-and-Whisker plots, and histograms can be used to organize the data for a single species. **This question is not on the worksheet.**	

	Have groups display their graphs and discuss differences among the graphs. Which one best displays the data, why? Do the show similar shapes? Why or why not? **Do not** change slides until you have finished discussing the groups' displays. The next three slides show what participants' graphs could have looked like.	
	(slide 12) **Stem-and-Leaf Plot** Lengths of Gray Whales 1 | 5 7 7 8 8 8 8 9 9 9 9 2 | 0 0 1 2 2 2 2 2 2 3 3 3 4 4 4 4 4 4 5 6 6 7 7 7 8 8 8 8 8 9 9 9 9 3 | 0 1 1 2 5 Key: 1 | 5 = 15 feet Does anyone have questions? Was yours different? Why?	
	(slide 13) **Box-and-Whiskers Plot** Lengths of Gray Whales  The minimum value is 15 feet. The maximum value is 35 feet. The value of the lower quartile is 20 feet. The median is 24 feet. The value of the upper quartile is 28 feet. Does anyone have questions? Was yours different? Why?	



(slide 14) **Histogram**
There are 12 whales with a length that is greater than or equal to 10 feet, but less than 20 feet.
There are 33 whales with a length that is greater than or equal to 20 feet, but less than 30 feet.
There are 5 whales with a length that is greater than or equal to 30 feet, but less than 40 feet.

Does anyone have questions? Was yours different? Why?



(slide 15) **Whale of a Time**
What information/misinformation does each of the graphs convey?
Stem-and-leaf plots do show all of the data points, but depending on the nature of the data they may hide the gaps.
For example:

Lengths of Gray Whales

1	5 7 7 8 8 8 8 8 9 9 9 9
3	0 1 1 2 5

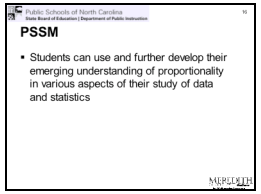
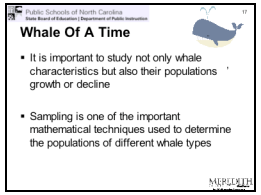
Key 1 | 5 = 15 feet

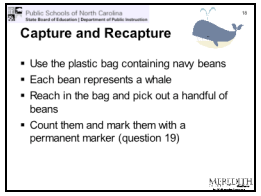
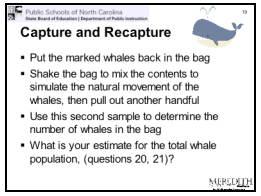
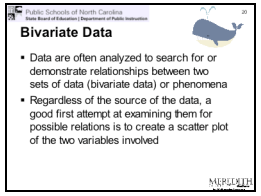
In this example, if no gray whales were 20-29 feet in length, the twenties are not represented in the plot.

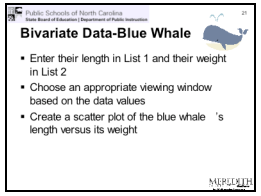
Box-and-whisker plots show ranges, not specific data.
Histograms show spread but hide specific data. Histograms could skew the picture of the data by choosing different scales for the lengths or frequency.

What does the variability in the each representation convey about the data?
The stem-and-leaf plot shows that most whales were 20 feet in length to 29 feet. Though as with any creature, a few are shorter than the rest and a few are longer than the rest. This could be due to the age of the whales.

The box-and-whiskers plot shows that the lengths are pretty evenly spread, with 50% of whales from 20 to 28 feet in length. Again the range could be due to the age and gender of the whales. There may have been a few young whales in the group and perhaps about the same number of

	older whales, with a few older female whales in the group. The histogram shows that the majority of the whales are 20-30 feet in length. About 1/3 are shorter and even less than that are longer. Among the whales that were measured, none are over 30 feet and none are less than 10 feet. They all show how the whale lengths fall within a normal range of lengths (no whales under 10 feet long and no whales more than 40 feet). According to http://www.whaletimes.org/whagray.htm, at birth whales are approximately 15 feet in length and female whales can grow to 45 feet in length while male whales are slightly smaller. **Also note, responding to variability is different than responding to an average, in that one number is not sufficient.**	
	(slide 16) **PSSM** Read the following quote from NCTM's Principles and Standards of School Mathematics (PSSM). *Students can use and further develop their emerging understanding of proportionality in various aspects of their study of data and statistics.* *PSSM, 2000*	
	(slide 17) **Whale of a Time** *It is important to study not only whale characteristics but also their populations' growth or decline.* *Sampling is one of the most important mathematical techniques used to determine the populations of different whale types* (since a census is not a reasonable goal) Many whale population estimates are based on observation. The National Marine Fisheries Service has clear rules for observing and counting whales. In the next activity, we will simulate sampling.	

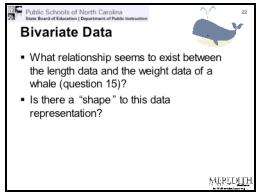
	(slide 18) **Capture and Recapture** MATERIALS: Navy Beans, Bag to hold beans, Permanent Marker The purpose this activity is to model the method used to estimate the population of wildlife. *Each group will be given a plastic bag containing navy beans. For the purposes of this activity, each bean represents a whale.* *Reach in the bag and pick out a handful of beans. Count them and mark them with a permanent marker (question 19).*	
	(slide 19) **Capture and Recapture** *Put the marked whales back in the bag. Shake the bag to mix the contents to simulate the natural movement of the whales, then pull out another handful* *Use this second sample to determine the number of whales in the bag. What is your estimate for the total whale population (questions 20, 21)?* Draw out another handful of whales. Count the total number of whales in the sample and record your answer (#20). Now count the number of marked whales in this sample and record that number (#20). Then answer question #21. Allow groups to share their sample results, their population estimates, and their reasoning behind the estimates (what proportion and relationships they used).	
	(slide 20) **Bivariate Data** *Data are often analyzed to search for or demonstrate relationships between two sets of data (bivariate data) or phenomena. Regardless of the source of the data, a good first attempt at examining them for possible relations is to create a scatter plot of the two variables involved.* Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc. We can analyze different characteristics within one whale type using a scatter plot.	



(slide 21) **Bivariate Data – Blue Whale**

- *Enter their length in List 1 and their weight in List 2.*
- *Choose an appropriate viewing window based on the data values.*
- *Create a scatter plot of the blue whale's length versus its weight.*

Going back to problem 15, create a scatter plot of the blue whale's length versus its weight. Enter the length in list 1 and the weight in list 2 within a calculator. Choose an appropriate viewing window based on the data values.



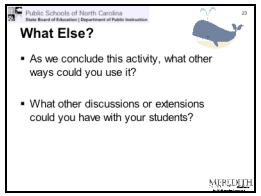
(slide 22) **Bivariate Data**

- *What relationship seems to exist between the length data and weight data of a whale (question 15)?*

What do you conclude about the relationship between the length and the weight of the whale?

- *Is there a "shape" to this data representation?*

In this case "shape" refers to a possible path through the data points.
We will finish #16, 17 in the next module, as time permits.

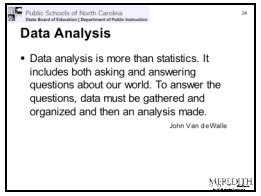


(slide 23) **What Else?**

- *As we conclude this activity, what other ways could you use it?*
- *What other discussions or extensions could you have with your students?*

Suggest comparing two or more sets of uncorrelated univariate data, such as lengths of blue whales and lengths of gray whales. These could easily be compared with side-by-side graphs.

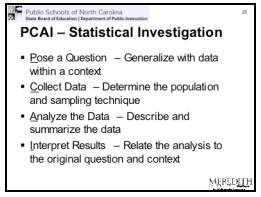
Or some non-examples of bivariate data could be created and discussed as well.
For example, the set of data points (12,67), (12,70), (13,55), (13,56), (14,39), (14,33), (15,22), (15,18), (15,17) seem to follow a pattern. However, if they represent the ordered pairs of (number of pairs of shoes, seconds breath held) one might incorrectly conclude that if you own fewer shoes you will be able to hold your breath for longer. This could be used to highlight that data analysis should not occur without regard to context and should be carried out to answer meaningful questions.



(slide 24) **Data Analysis**
Data analysis is more than statistics. It includes both asking and answering questions about our world. To answer the questions, data must be gathered and organized and then an analysis made.

Van de Walle, J. A (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Learning Inc.

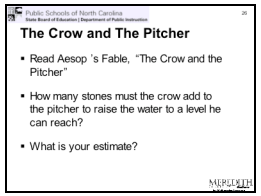
Ask participants to reflect again on this quote. What types of data might interest students?



(slide 25) **PCAI – Statistical Investigation**

- *Pose a Question – Generalize with data within a context*
- *Collect Data – Determine the population and sampling technique*
- *Analyze the Data – Describe and summarize the data*
- *Interpret Results – Relate the analysis to the original question and context*

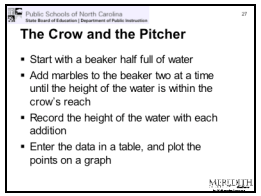
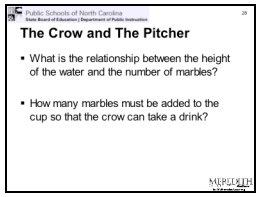
Ask a volunteer to read the bullets on the slide. Note to participants that this PCAI model of statistical investigation is a part of the new K-5 curriculum and encourages the use of a context when working with data. It is common for mathematics teachers to focus on the mechanics of data analysis (how to calculate the mean, etc.) without regard to the context. This however does not suffice in preparing our students to understand the uses and misuses of data in everyday life. The PCAI model is one example of an approach to working with data in a meaningful way within a context.

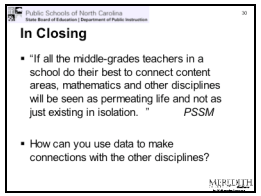


(slide 26) **The Crow and the Pitcher**
MATERIALS: Crow and the Pitcher handout, Beakers, Marbles, Water, Rulers, Chart Paper, Markers

Read Aesop's Fable, "The Crow and the Pitcher."
Read from the handout.

How many stones must the crow add to the pitcher to raise the water level within 5 cm of the top of the pitcher?

	What is your estimate? Participants should give a reasonable estimate of the number of stones that will be needed.	
	(slide 27) **The Crow and the Pitcher** ▪ *Start with a beaker half full of water.* ▪ *Add marbles to the beaker two at a time until the height of the water is within the crow's reach.* ▪ *Record the height of the water with each addition.* ▪ *Enter the data in a table, and plot the points on a graph.* Be sure there is enough water in the cup so that the added marbles are completely submerged. Having participants work in groups, allow them time to experiment and record their data.	
	(slide 28) **The Crow and the Pitcher** *What is the relationship between the height of the water and the number of marbles?* Answers may vary, but the big idea is that if the number of marbles in the beaker increases, then the height of the water increases. *How many marbles must be added to the cup so that the crow can take a drink?* If participants did not have enough marbles in their sample to answer the last question, ask them to make estimates based on their data. This activity will be revisited in Module 6 when participants will look more at the line of best fit and slope. Ask participants to keep their work in a safe place. (Another suggestion would be for the trainer to collect the work and hold onto it until you meet again for the next module.)	
	(slide 29) **Where does it all fit?** *Where do these activities in the Standards?* *Where do these activities in the Big Ideas?* Ask participants to work in their table groups to look at both the Essential Standards and the Big Ideas. Then ask groups to share their ideas. Some connections may include the following. • A variety of representations (including tables, charts, graphs, number lines, expressions, equations, and inequalities) can be used to illustrate mathematical relationships, to model	

	mathematical situations, or to describe and generalize patterns • The understanding of proportional reasoning and rates of change promotes algebraic thinking and development • Collection, analysis, and interpretation of univariate data are used to make decisions and solve problems • Bivariate data may be displayed and then analyzed within the rectangular coordinate plane, where a linear equation may be a good model for the relationship between the two attributes • Statistical investigations are completed through a process that includes posing a problem, collecting and analyzing data, and interpreting results	
	(slide 30) **In Closing** In this module, we made connections to other disciplines (explicitly to science and literature). *"If all the middle-grades teachers in a school do their best to connect content areas, mathematics and other disciplines will be seen as permeating life and not as just existing in isolation." PSSM* *How can you use data to make connections with other disciplines?* Participants should write down their thoughts. After a couple of minutes, solicit answers from participants.	
	(slides 31-34) **Closing and credit slides**	