Cluster 2: Explore multiplicative comparison, area and perimeter, factors, and multiples
Duration: 3-4 weeks

Content Standards
This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Represent and solve problems involving multiplication and division.
NC.4.OA.1
Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

Gain familiarity with factors and multiples.
NC.4.OA.4
Find all factor pairs for whole numbers up to and including 50 to:
- Recognize that a whole number is a multiple of each of its factors.
- Determine whether a given whole number is a multiple of a given one-digit number.
- Determine if the number is prime or composite.

NC.4.MD.3
Solve problems with area and perimeter.
- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters with a fixed perimeter and varying areas.
- ~~Apply the area and perimeter formulas for rectangles in real world and mathematical problems.~~

Use the four operations with whole numbers to solve problems.
NC.4.OA.3
Solve two-step word problems involving the four operations with whole numbers.
- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for unknown quantity.

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

What is the mathematics?
In third grade, students focus on equal group and array multiplicative situations. In fourth grade, multiplicative thinking is expanded to include multiplicative comparison. Multiplicative comparison problems describe the relationship between two quantities, in which one is a multiple of the other (times as many, times more/less than). Data and scaled bar graphs from the previous cluster can be used as a context to represent and discuss both additive and multiplicative comparisons and to help students distinguish between the two. In fourth grade, students continue to develop the concept of area based on the array model and use that model to explore factors and multiples.

- Students will investigate and explain the difference between multiplicative comparisons and additive comparisons (ex: What is the difference between saying 3 times as many/ 3 times less than and 3 more/3 less than?).
- Students will explore multiplicative relationships through multiple contexts and models (ex: area model, tape diagram, scale bar graph, measurement situations, money situations).
- Students will solve both unknown product and unknown factor situations (ex: Keenan is 4 feet tall, the tree he planted is now 5 times taller than he is. How tall is the tree?; Keenan is 4 feet tall. The tree is 24 feet. How many times taller is the tree than Keenan? In both examples, students should draw a diagram to show the relationship between Keenan's height and the height of the tree.)
- Students will explore finding factors using the array/area model (ex: Find all the rectangles that can be made using exactly 24 tiles. For rectangles with an area of 24, the side lengths of the rectangles are factors of 24).
- Students will verify that some numbers can be made into more than one rectangular array (composite numbers) and some numbers can only be represented by rectangular arrays with 1 row (prime numbers); then use that understanding to define prime and composite numbers.
- Students will investigate and discover that a whole number is a multiple of each of its factors (ex: Make a rectangle to show that 4 is a factor of 24? How do you know?; How many hops of 6 on a number line does it take to get to 24? So, 24 is 4 times as many as six. What other equal hops could you make to reach 24?).
- Students will explore and compare fixed areas and fixed perimeters and formulate conclusions about the relationship between area and perimeter.

Important Considerations:
- The work in this cluster builds off the 3rd grade focus of multiplication as repeated addition and equal groups and includes a new interpretation in 4th grade of multiplication as comparison.
- Multiplicative comparison situations in this cluster should be limited to comparisons within 100 focusing first on building foundational language and conceptual understanding of this new interpretation.
- Placing this cluster early in the year will allow for time to review multiplication facts within 100 to further develop fluency.
- When exploring multiplicative comparison, the phrases *how many times more than/less than, how many times fewer than*, and *times as many* help connect the understanding that the comparison is based on one set being a multiplier of the other. Using a tape diagram (also known as bar model) can help develop this idea. (ex: If four times as many 3rd grade students ride the bus to school compared to the 10 that walk to school, how many students ride the bus?)

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- Many rich contexts can be used to practice multiplicative comparison (ex. Building off data collection from cluster 1, if twice as many people ride in cars to school compared to walking; review customary measurement from grade 3 to discover that a quart is four times as much as a cup and twice as much as a pint; In pattern-finding, the number of blocks in a shape pattern can be 3 times the term number or the number of eyes in 6 dogs is twice as many as the number of eyes in 3 dogs).
- Multiplicative comparison can also be used in area and perimeter investigations (ex: A rectangle is 2 tiles wide. It is 5 times as long as it is wide. Create a model to find the rectangle's length; A dog kennel is 5 feet wide. It is 4 times as long as it is wide. What is the perimeter of the kennel?).
- There are many misconceptions related to the concepts of area and perimeter. It is important that formulas are not preceded by the understanding of each unique measure. Provide hands-on experiences, where students manipulate and measure shapes to discover students that it is possible to change the area of a figure without changing its perimeter.
- Students will investigate and explain that it is possible to have different rectangles with the same perimeter but different areas and vice versa (ex: Students investigate how many different rectangles can be made with 36 tiles. Find and record the perimeter of the rectangle. Also, using 24 cm of string, students investigate the different rectangles that can be made with the fixed perimeter. Find and record the area.).