General Descriptions of NAEP Levels of Complexity
(NAEP = National Assessment of Educational Progress)

"Mathematical complexity attempts to focus on the cognitive demands of the assessment question. Each level of complexity includes aspects of knowing and doing mathematics, such as reasoning, performing procedures, understanding concepts, or solving problems. The levels of complexity form an ordered description of the demands an item may make on a student. Items at the low level of complexity, for example, may ask a student to recall a property. At the moderate level, an item may ask the student to make a connection between two properties; at the high level, an item may ask a student to analyze the assumptions made in a mathematical model."
(http://nces.ed.gov/nationsreportcard/mathematics/whatmeasure.asp)

Levels of complexity relate to the demands on thinking and deal with what the student is asked to do in a task. The assumption is that the student is familiar with the mathematics of the task and does not relate to how a student might complete the task. For example, in an area problem one student might apply the formula from memory while another student might draw a model.

Low Complexity

This category relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a low-complexity category:

- Recall or recognize a fact, term, or property
- Recognize an example of a concept
- Compute a sum, difference, product, or quotient
- Recognize an equivalent representation
- Perform a specified procedure
- Evaluate an expression in an equation or formula for a given variable
- Solve a one-step word problem
- Draw or measure simple geometric figures
- Retrieve information from a table or graph

Example 1

Low Complexity Source: 1996 NAEP 4M9 #1 Grade 4 Percent correct: 50% Number Properties and Operations: Number sense No calculator

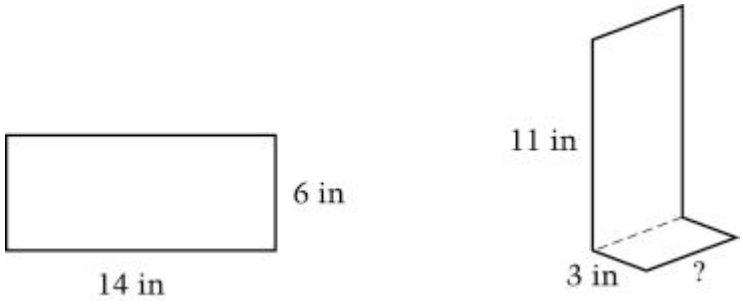
How many fourths make a whole? Answer: _____ Correct answer: 4

Rationale: This item is of low complexity since it explicitly asks students to recognize an example of a concept (four-fourths make a whole).

Low Complexity examples continued

Example 2

Low Complexity Source: 2005 NAEP 4M12 #12 Grade 4 Percent correct: 54% Geometry: Transformations of shapes No calculator



A piece of metal in the shape of a rectangle was folded as shown above. In the figure on the right, the "?" symbol represents what length? A. 3 inches B. 6 inches C. 8 inches D. 11 inches

Correct answer: B

Rationale: Although this is a visualization task, it is of low complexity since it requires only a straightforward recognition of the change in the figure. Students in the 4th grade are expected to be familiar with sums such as 11 + 3, so this does not increase the complexity level for these students.

Example 3

Low Complexity Source: 2005 NAEP 8M12 #17 Grade 8 Percent correct: 54% Algebra: Algebraic representations No calculator

x	0	1	2	3	10
y	–1	2	5	8	29

Which of the following equations represents the relationship between x and y shown in the table above? A. $y = x^2 + 1$ B. $y = x + 1$ C. $y = 3x - 1$ D. $y = x^2 - 3$ E. $y = 3x^2 - 1$

Correct answer: C

Rationale: This item would be at the moderate level if it were written as follows, "Write the equation that represents the relationship between x and y." In generating the equation students would first have to decide if the relationship was linear.

Moderate Complexity

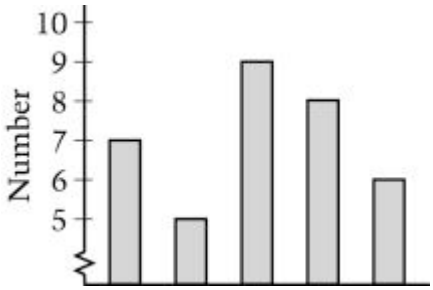
Items in the moderate-complexity category involve more flexibility of thinking and choice among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do, using informal methods of reasoning and problem-solving strategies, and to bring together skill and knowledge from various domains.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a moderate-complexity category:

- Represent a situation mathematically in more than one way
- Select and use different representations, depending on the situation
- Solve a word problem requiring multiple steps
- Compare figures or statements
- Provide justification for steps in a solution process
- Interpret a visual representation
- Extend a pattern
- Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps
- Formulate a routing problem, given data and conditions
- Interpret a simple argument

Example 1:

Moderate Complexity Source: 2005 NAEP 4M12 #11 Grade 4 Percent correct: 52% Data Analysis, Statistics, and Probability: Data representation No calculator

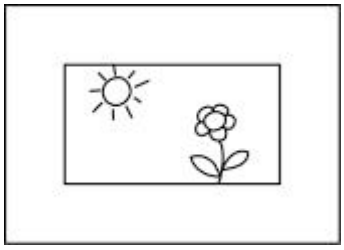


Jim made the graph above. Which of these could be the title for the graph?
A. Number of students who walked to school on Monday through Friday
B. Number of dogs in five states
C. Number of bottles collected by three students
D. Number of students in 10 clubs Correct answer: A

Rationale: Students must analyze the graph and the choices for a title and eliminate choices because of knowledge of dogs and clubs and the structure of the graph (five sets of data) in order to choose an appropriate title for the graph.

Example 2

Moderate Complexity Source: 2005 NAEP 8M3 #3 Grade 8 Percent correct: 44% (full credit), Measurement: Measuring physical attributes 13% (partial credit) No calculator, ruler provided



The figure above shows a picture and its frame. In the space below, draw a rectangular picture 2 inches by 3 inches and draw a 1-inch wide frame around it.

Rationale: Students must plan their drawing, decide whether to begin with the inside or outside rectangle, and determine how the other rectangle is related to the one chosen. Often creating a drawing that satisfies several conditions is more complex than describing a given figure.

Example 3

Moderate Complexity Source: 2005 NAEP 8M3 #10 Grade 8 Percent correct: 34% Algebra: Patterns, relations, and functions No calculator

In the equation $y = 4x$, if the value of x is increased by 2, what is the effect on the value of y?
A. It is 8 more than the original amount
B. It is 6 more than the original amount
C. It is 2 more than the original amount
D. It is 16 times the original amount
E. It is 8 times the original amount Correct answer: A

Rationale: This item is of moderate complexity because it involves more flexibility and a choice of alternative ways to approach the problem rather than a low complexity level which more clearly states what to be done. At grade 8, students have not learned a procedure for answering this type of question.

Example 4

Moderate Complexity Source: 2005 NAEP B3M3 Grade 12 Percent correct: 22% Number Properties and Operations: Number operations No calculator

The remainder when a number n is divided by 7 is 2. Which of the following is the remainder when $2n + 1$ is divided by 7?
A. 1 D. 4
B. 2 E. 5
C. 3 Correct answer: E

Rationale: Although the problem could be approached algebraically (n = 7m + 2, for some whole number m, and 2n + 1= 2(7m + 2) + 1 or 14 m + 5, so the remainder is 5), students can solve the problem by using a value for n that satisfies the condition that it has a remainder of 2 when divided by 7. If the students were asked to justify their solution algebraically, then this would be an item of high complexity.

High Complexity

High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in an abstract and sophisticated way.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a high-complexity category:

- Describe how different representations can be used for different purposes
- Perform procedures having multiple steps and multiple decision points
- Analyze situations and differences between procedures and concepts
- Generalize a pattern
- Formulate an original problem, given a situation
- Solve a novel problem
- Describe, compare, and contrast solution methods
- Formulate a mathematical model for a complex situation
- Analyze the assumptions made in a mathematical model
- Analyze a deductive argument
- Provide a mathematical justification

Example 1

High Complexity Source: 2003 NAEP 4M7 #20 Grade 4 Percent correct: 3% (extended), Algebra: Patterns, relations, and functions 6% (satisfactory), 13% (partial), 27% (minimal) Calculator available

The table below shows how the chirping of a cricket is related to the temperature outside. For example, a cricket chirps 144 times each minute when the temperature is 76°.

Number of Chirps Per Minute	Temperature
144	76°
152	78°
160	80°
168	82°
176	84°

What would be the number of chirps per minute when the temperature outside is 90° if this pattern stays the same?

Answer: _____ Explain how you figured out your answer.

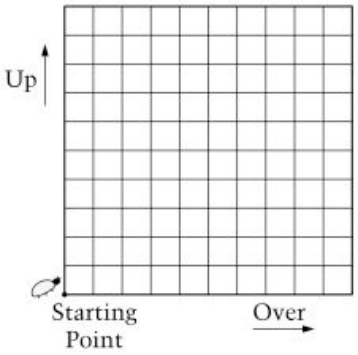
Correct answer: 200

Rationale: To receive full credit for this item, students must give the correct number of chirps and explain that for every 2-degree rise in the temperature, the number of chirps increases by eight. The item requires creative thought for students at this grade as well as planning a solution strategy. Additionally, it requires a written justification of their answer, more than just showing work.

Example 2

High Complexity Source: 2005 NAEP 8M4 #11 Grade 8 Percent correct: 12% (full credit), Algebra: Patterns, relations, and functions 24% (partial credit) No calculator

If the grid in Question 10 [the previous question] were large enough and the beetle continued to move in the same pattern [over 2 and up 1], would the point that is 75 blocks up and 100 blocks over from the starting point be on the beetle's path?



Give a reason for your answer.

Rationale: Students must justify their yes or no answer by using the concept of slope showing that moving over 2 and up 1 repeatedly would result in the beetle being at a point 100 blocks over and 50 blocks up. This requires analysis of the situation as well as a mathematical explanation of the thinking. Since it is not realistic to extend the grid, students are expected to generalize about the ratio.

"The assumption underlying these levels is that what a student does with the mathematics he or she learns is important. If students are asked only to recall information or to perform routine procedures instructionally, they will not likely be able to solve complex problems relate to the same mathematics."

NCTM, *Grades 6-8 Mathematics Sampler*, p. 225

Information in this handout comes from the NAEP 2009 Mathematics Framework and the *Grades 6-8 Mathematics Sampler* published by the National Council of Teachers of Mathematics, pages 224-225. For more information, view the NAEP 2009 Mathematics Framework: http://www.nagb.org/publications

Level Comparison Page

Low Complexity:

- Recall or recognize a fact, term, or property
- Recognize an example of a concept
- Compute a sum, difference, product, or quotient
- Recognize an equivalent representation
- Perform a specified procedure
- Evaluate an expression in an equation or formula for a given variable
- Solve a one-step word problem
- Draw or measure simple geometric figures
- Retrieve information from a table or graph

Moderate/High Complexity - Not Low (for purposes of K-2 sort):

- Represent a situation mathematically in more than one way
- Select and use different representations, depending on the situation
- Solve a word problem requiring multiple steps
- Compare figures or statements
- Provide justification for steps in a solution process
- Interpret a visual representation
- Extend a pattern
- Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps
- Formulate a routing problem, given data and conditions
- Interpret a simple argument
- Describe how different representations can be used for different purposes
- Perform procedures having multiple steps and multiple decision points
- Analyze situations and differences between procedures and concepts
- Generalize a pattern
- Formulate an original problem, given a situation
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- Describe, compare, and contrast solution methods
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- Analyze the assumptions made in a mathematical model
- Analyze a deductive argument
- Provide a mathematical justification

Task A:

Manipulatives or Tools Available: counters, Unifix cubes, base ten blocks, paper & pencils

Mr. Scott's sons picked 18 apples. Billy picked 15 apples. Mr. Scott needs ten apples to make a pie. How many pies can Mr. Scott bake?

Task B:

Manipulatives or Tools Available: Four or five containers, water, sand, or pellets and a variety of cups or scoops

Which container holds the most?
Which container holds the least?
How do you know?
Explain with pictures, words, or numbers.

Task C:

Manipulatives or Tools Available: none

Draw a box around the smallest number:

359 953 535 395

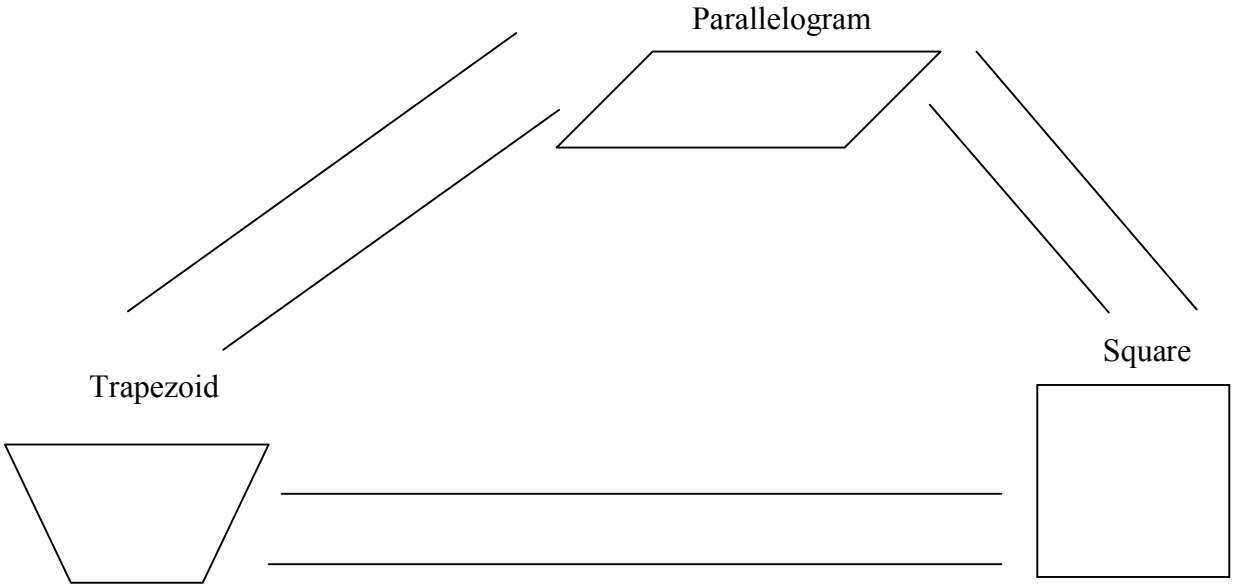
Task D:

Manipulatives or Tools Available: counting jar filled with red and blue cubes (number determined by grade level)

Count the objects in the jar to find out how many cubes are in it. Then make a set that has that same amount of objects in it and record what you found out.

Task E:

Manipulatives or Tools Available: None



Describe two ways the shapes are mathematically alike and two ways they are mathematically different.

Task F:

Manipulatives or Tools Available:

Mr. Nolan's class estimated the number **o**f beans in this jar. These are their estimates:

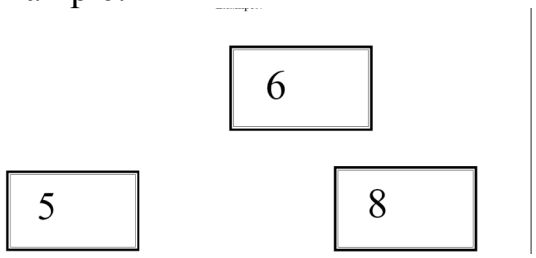
18	////
22	////
26	///
30	//
43	////
52	///

Make a line plot using this information.

Task G:

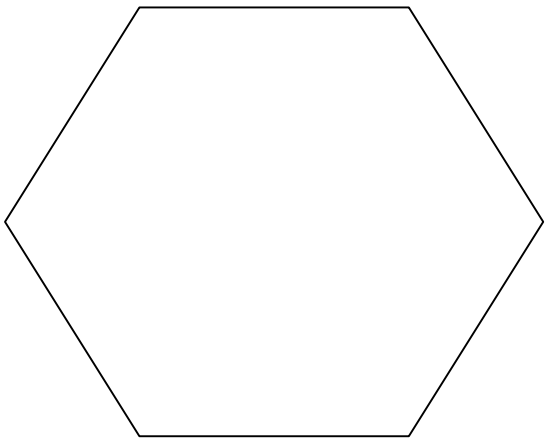
Manipulatives or Tools Available:

Teacher gives students a bag of counters or cubes and asks them to make sets to match the numbers on cards.
Example:



Task H:

Manipulatives or Tools Available: pattern blocks
Students are provided with 15 or 20 assorted pattern blocks. Students are asked to fill (cover) a given shape with pattern blocks.



Task I:

Manipulatives or Tools Available: place value blocks

Write 412 in three different ways.
Identify the place for each digit.
What value is each digit in that number?

Task J:

Manipulatives or Tools Available: Table listing students names and the number of pockets each child has today, coins if needed.

Using the class graph of number of pockets for each student, determine how many pennies the class gets if each pocket gets a penny. How could the class trade the pennies for a fewer number of coins?

Task K:

Manipulatives or Tools Available: Ten Black Dots book, paper & pencil

Read the book, Ten Black Dots to the class. Explain to the class we are going to make our own class book but first need to know if we have enough dots. How many dots do we need to make our class book?

Task L:

Manipulatives or Tools Available: each child receives 6 two-color counters and a recording sheet

Model dropping 6 counters and recording the red and yellow combinations. Allow students to drop the counters to find out all of the possible combinations of 6 two-colored counters.

Task M:

Manipulatives/Tools Available: Counters, cubes, grid paper, base-ten blocks

Solve this problem in two different ways:

32 - 17

After each way, write about how you did it. Be sure to include:

- what materials, if any, you used to solve this problem
- how you solved it
- an explanation of your thinking as you solved it

First Way:

Second Way:

Task N:

Manipulatives/Tools Available: none

Mrs. Jones' class worked the following problem. After the students worked the problem Mrs. Jones put these solutions on the overhead/document camera. She asked the students to discuss how the strategies are alike and how they are different. What did the students say?

There were 77 snowflakes on the birdbath. Then some more snowflakes fel
Now there are 92 snowflakes on the birdbath. How many more snowflakes fell?