Identifying Characteristics of Mathematical Tasks

Task 1:
You are carpeting a room that is 3 yards long and 4 yards wide. How many square yards of carpeting do you need to purchase?

Task 2:
Our class will raise chickens for the science fair and can use 24 feet of fencing to build a rectangular pen
- ✓ How long are the sides that give the chickens as much room as possible?
- ✓ How long are the sides if only 16 feet of fencing is used?
- ✓ How would you determine the pen with the most room for any amount of fencing?

Look at the two tasks:
How would you go about solving them?

What would be a possible learning target you would have for selecting these tasks?

Which of the two tasks requires students to do more in-depth thinking and causes them to be more intellectually engaged?

Comparisons
How are the two tasks alike?

How are they different?

For each task:

How/when would you implement this task?

What is the role of the teacher and students?

What skills were employed during the task?

What kinds of information about a student's thinking can be gathered through each task?

General Descriptions of NAEP Levels of Complexity
(NAEP = National Assessment of Educational Progress)

"Mathematical complexity attempts to focus on the cognitive demands of the assessment question. Each level of complexity includes aspects of knowing and doing mathematics, such as reasoning, performing procedures, understanding concepts, or solving problems. The levels of complexity form an ordered description of the demands an item may make on a student. Items at the low level of complexity, for example, may ask a student to recall a property. At the moderate level, an item may ask the student to make a connection between two properties; at the high level, an item may ask a student to analyze the assumptions made in a mathematical model."

(http://nces.ed.gov/nationsreportcard/mathematics/whatmeasure.asp)

Levels of complexity relate to the <u>demands on thinking</u> and deal with what the student is asked to do in a task. The assumption is that the student is familiar with the mathematics of the task and does not relate to how a student might complete the task. For example, in an area problem one student might apply the formula from memory while another student might draw a model.

Low Complexity

This category relies heavily on the <u>recall and recognition</u> of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a low-complexity category:
• Recall or recognize a fact, term, or property
• Recognize an example of a concept
• Compute a sum, difference, product, or quotient
• Recognize an equivalent representation
• Perform a specified procedure
• Evaluate an expression in an equation or formula for a given variable
• Solve a one-step word problem
• Draw or measure simple geometric figures
• Retrieve information from a table or graph

Example 1

Low Complexity Source: 1996 NAEP 4M9 #1 Grade 4 Percent correct: 50% Number Properties and Operations: Number sense No calculator

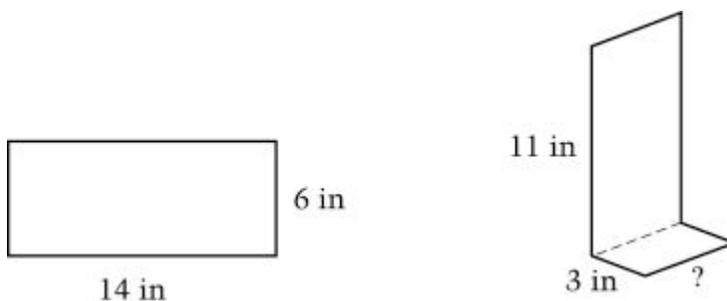How many fourths make a whole? Answer: _____   Correct answer: 4

Rationale: This item is of low complexity since it explicitly asks students to recognize an example of a concept (four-fourths make a whole).

Low Complexity examples continued

Example 2

Low Complexity Source: 2005 NAEP 4M12 #12 Grade 4 Percent correct: 54% Geometry: Transformations of shapes No calculator

A piece of metal in the shape of a rectangle was folded as shown above. In the figure on the right, the "?" symbol represents what length? A. 3 inches B. 6 inches C. 8 inches D. 11 inches Correct answer: B

Rationale: Although this is a visualization task, it is of low complexity since it requires only a straightforward recognition of the change in the figure. Students in the 4th grade are expectbe familiar with sums such as 11 + 3, so this does not increase the complexity level for these students.

Example 3

Low Complexity Source: 2005 NAEP 8M12 #17 Grade 8 Percent correct: 54% Algebra: Algebraic representations No calculator

x	0	1	2	3	10
y	−1	2	5	8	29

Which of the following equations represents the relationship between x and y shown in the table above? A. $y = x^2 + 1$ B. $y = x + 1$ C. $y = 3x - 1$ D. $y = x^2 - 3$ E. $y = 3x^2 - 1$

Correct answer: C

Rationale: This item would be at the moderate level if it were written as follows, "Write the equation that represents the relationship between x and y." In generating the equation students would first have to decide if the relationship was linear.

Moderate Complexity

Items in the moderate-complexity category involve <u>more flexibility of thinking and choice</u> among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do, using informal methods of reasoning and problem-solving strategies, and to bring together skill and knowledge from various domains.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a moderate-complexity category:
- Represent a situation mathematically in more than one way
- Select and use different representations, depending on the situation
- Solve a word problem requiring multiple steps
- Compare figures or statements
- Provide justification for steps in a solution process
- Interpret a visual representation
- Extend a pattern
- Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps
- Formulate a routing problem, given data and conditions
- Interpret a simple argument

Example 1:

Moderate Complexity Source: 2005 NAEP 4M12 #11 Grade 4 Percent correct: 52% Data Analysis, Statistics, and Probability: Data representation No calculator
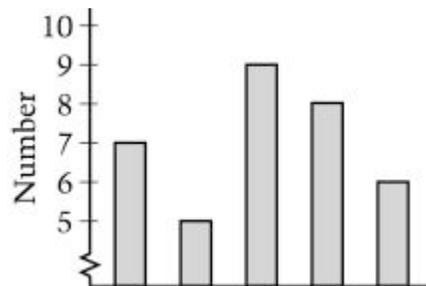

Jim made the graph above. Which of these could be the title for the graph?
A. Number of students who walked to school on Monday through Friday
B. Number of dogs in five states
C. Number of bottles collected by three students
D. Number of students in 10 clubs   Correct answer: A

Rationale: Students must analyze the graph and the choices for a title and eliminate choices because of knowledge of dogs and clubs and the structure of the graph (five sets of data) in order to choose an appropriate title for the graph.

Example 2

Moderate Complexity Source: 2005 NAEP 8M3 #3 Grade 8 Percent correct: 44% (full credit), Measurement: Measuring physical attributes 13% (partial credit) No calculator, ruler provided
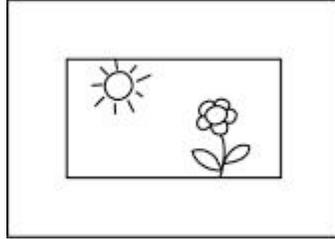
The figure above shows a picture and its frame. In the space below, draw a rectangular picture 2 inches by 3 inches and draw a 1-inch wide frame around it.

Rationale: Students must plan their drawing, decide whether to begin with the inside or outside rectangle, and determine how the other rectangle is related to the one chosen. Often creating a drawing that satisfies several conditions is more complex than describing a given figure.

Example 3

Moderate Complexity Source: 2005 NAEP 8M3 #10 Grade 8 Percent correct: 34% Algebra: Patterns, relations, and functions No calculator

In the equation $y = 4x$, if the value of x is increased by 2, what is the effect on the value of y?
A. It is 8 more than the original amount
B. It is 6 more than the original amount
C. It is 2 more than the original amount
D. It is 16 times the original amount
E. It is 8 times the original amount   Correct answer: A

Rationale: This item is of moderate complexity because it involves more flexibility and a choice of alternative ways to approach the problem rather than a low complexity level which more clearly states what to be done. At grade 8, students have not learned a procedure for answering this type of question.

Example 4

Moderate Complexity Source: 2005 NAEP B3M3 Grade 12 Percent correct: 22% Number Properties and Operations: Number operations No calculator

The remainder when a number *n* is divided by 7 is 2. Which of the following is the remainder when 2*n* + 1 is divided by 7?

A. 1   D. 4

B. 2   E. 5

C. 3   Correct answer: E

Rationale: Although the problem could be approached algebraically (n = 7m + 2, for some whole number m, and 2n + 1= 2(7m + 2) + 1 or 14 m + 5, so the remainder is 5), students can solve the problem by using a value for n that satisfies the condition that it has a remainder of 2 when divided by 7. If the students were asked to justify their solution algebraically, then this would be an item of high complexity.

High Complexity

High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in an abstract and sophisticated way.

Below are some examples given in the NAEP 2005 Framework of the demands made by items in a high-complexity category:
- Describe how different representations can be used for different purposes
- Perform procedures having multiple steps and multiple decision points
- Analyze situations and differences between procedures and concepts
- Generalize a pattern
- Formulate an original problem, given a situation
- Solve a novel problem
- Describe, compare, and contrast solution methods
- Formulate a mathematical model for a complex situation
- Analyze the assumptions made in a mathematical model
- Analyze a deductive argument
- Provide a mathematical justification

Example 1

High Complexity Source: 2003 NAEP 4M7 #20 Grade 4 Percent correct: 3% (extended), Algebra: Patterns, relations, and functions 6% (satisfactory), 13% (partial), 27% (minimal) Calculator available

The table below shows how the chirping of a cricket is related to the temperature outside. For example, a cricket chirps 144 times each minute when the temperature is 76°.

Number of Chirps Per Minute	Temperature
144	76°
152	78°
160	80°
168	82°
176	84°

What would be the number of chirps per minute when the temperature outside is 90° if this pattern stays the same?
Answer: _____ Explain how you figured out your answer.

Correct answer: 200

Rationale: To receive full credit for this item, students must give the correct number of chirps and explain that for every 2-degree rise in the temperature, the number of chirps increases by eight. The item requires creative thought for students at this grade as well as planning a solution strategy. Additionally, it requires a written justification of their answer, more than just showing work.

Example 2

If the grid in Question 10 [the previous question] were large enough and the beetle continued to move in the same pattern [over 2 and up 1], would the point that is 75 blocks up and 100 blocks over from the starting point be on the beetle's path?
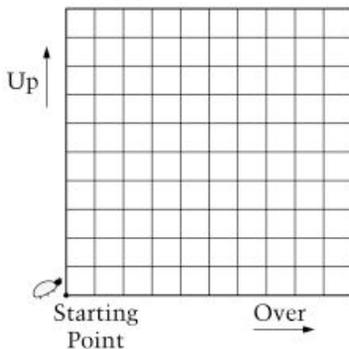
Give a reason for your answer.

Rationale: Students must justify their yes or no answer by using the concept of slope showing that moving over 2 and up 1 repeatedly would result in the beetle being at a point 100 blocks over and 50 blocks up. This requires analysis of the situation as well as a mathematical explanation of the thinking. Since it is not realistic to extend the grid, students are expected to generalize about the ratio.

"The assumption underlying these levels is that what a student does with the mathematics he or she learns is important. If students are asked only to recall information or to perform routine procedures instructionally, they will not likely be able to solve complex problems relate to the same mathematics."

NCTM, *Grades 6-8 Mathematics Sampler*, p. 225

Information in this handout comes from the NAEP 2009 Mathematics Framework and the *Grades 6-8 Mathematics Sampler* published by the National Council of Teachers of Mathematics, pages 224-225. For more information, view the NAEP 2009 Mathematics Framework: http://www.nagb.org/publications

TASK SORT

TASK A:

Find the smallest positive integer that has exactly 13 factors.

TASK B:

Your math class has a ratio of boys to girls of 2:3. If there are 8 boys in your math class, how many girls are there?

TASK C:

Create a set of data that includes 5 different temperatures. Your data must meet the following conditions:
- The sum of the highest and lowest temperatures must be 0°.
- The mean temperature must be –3°.

Explain how you know your data meet the two conditions.
*(Adapted from Mathematics Assessment Sampler, Grades 6 – 8, NCTM)*

TASK D:

At the end of the fourth day of their fund-raising campaign, the teachers at our school had raised $270 of the $360 they needed to reach their goal. Three teachers got into a debate about how to report their progress.
- Ms. Mendelez wanted to announce they were three-fourths of the way to their goal.
- Mr. Perk said six-eighths was a better description.
- Ms. Christie suggests two-thirds.

Which teacher do you agree with? Why?
How could the teacher you agreed with prove his/her case?

$270	**GOAL**
	$360
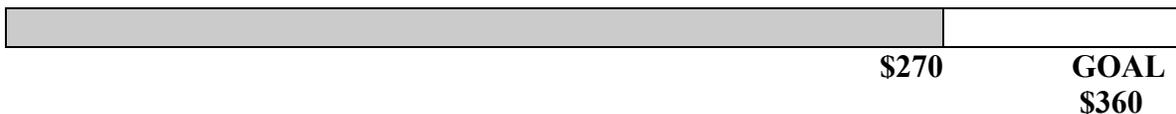
TASK E:

Match the property with the appropriate equation.

1. Commutative property of addition

2. Commutative property of multiplication

3. Associative property of addition

4. Associative property of multiplication

5. Identity property of addition

6. Identity property of multiplication

7. Inverse property of addition

8. Inverse property of multiplication

9. Distributive property

10. Zero property for multiplication

a. $r(s + t) = rs + rt$

b. $\boxed{} = 1$

c. $\boxed{}$

d. $\boxed{}$

e. $\boxed{}$

f. $\boxed{}$

g. $\boxed{}$ and $\boxed{}$

h. $\boxed{}$

i. $y + 0 = y$

j. $\boxed{}$

TASK F:

Write and solve a proportion for each of the following:

17 is what percent of 68?
What is 15% of 60?
8 is 10% of what number?
24 is 25% of what number?
28 is what percent of 140?
What is 60% of 45?

TASK G:
If the area of a circle is 7,850 square inches, what is the radius of the circle?

TASK H:
If the height of a rectangular prism is doubled, how does the volume change? Explain your answer.

TASK I:
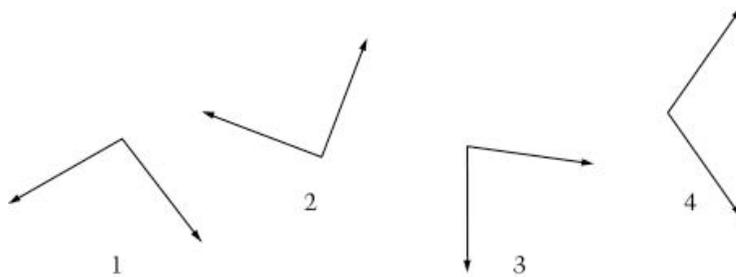Explain how each number below is different from all the others.

81   ☐   36   14

TASK J:
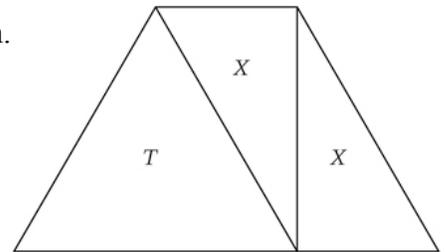Lists the angles shown below in order of measure from smallest to largest?
(Adapted from NAEP, Grade 8, 2005, Released Questions)

TASK K:
You will need pieces labeled *R*, *T*, and *X* to answer this question.
(From NAEP, Grade 8, 2009, Released Questions)

The figure above is made of one piece labeled *T* and two pieces labeled *X*. This figure has the same total area as

    A. one piece labeled *R*
    B. two pieces labeled *X*
    C. three pieces labeled *X*
    D. one piece labeled *R* and one piece labeled *T*

TASK L:
Andrew has one quarter, one dime, one nickel, and one penny. Two of the coins are in his left pocket and the other two coins are in his right pocket. The coins have been randomly placed in the two pockets. What is the probability that Andrew will be able to purchase a 30-cent candy bar with the two coins in his left pocket? Using the coins, explain your reasoning. *(Adapted from NAEP, Grade 8, 2009, Released Questions)*

TASK M:
Which of the following equations has the same solution as the equation $2x + 8 = 32$?
  A. $2x = 40$
  B. $x - 4 = 16$
  C. $x + 8 = 16$
  D. $2(x - 4) = 16$
  E. $2(x + 4) = 32$

(Adapted from NAEP, Grade 8, 2009, Released Questions)

TASK N:
Write two different equations that have the same solution as the equation $3x - 9 = 36$?

TASK O:
A certain even number is divisible by 9. This number is between 100 and 120. What is the number? *(From NAEP, Grade 8, 2009, Released Questions)*

TASK P:
The distance around a circular pond are shown below. From the snack bar, Jake notices an island in the center of the pond. Of the following, which is the best approximation of the distance from the snack bar to the center of the island?
(From NAEP, Grade 8, 2007, Released Questions)

  A. 16 yards
  B. 20 yards
  C. 32 yards
  D. 50 yards
 E. 64 yards
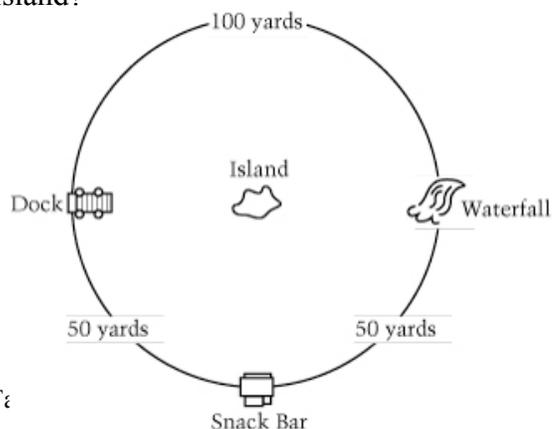
TASK Q:

The Millers are going to build a new one-story house. The floor of the house will be rectangular with a length of 30 feet and a width of 20 feet. The house will have a living room, a kitchen, two bedrooms, and a bathroom. *(From NAEP, Grade 8, 2009, Released Questions)*

In part (a) below create a floor plan that shows these five rooms by dividing the rectangle into rooms.

Your floor plan should meet the following conditions.

- Each one of the five rooms must share at least one side with the rectangle in part (a); that is, each room must have at least one outside wall.
- The floor area of the bathroom should be 50 square feet.
- Each of the other four rooms (not the bathroom) should have a length of at least 10 feet and a width of at least 10 feet.
- Be sure to label each room by name (living room, kitchen, bedroom, etc.) and include its length and width, in feet. (Do not draw any hallways on your floor plan.)

(a) Draw your floor plan on the figure below. Remember to label your rooms by name and include the length and width, in feet, for each room.


20 ft

30 ft
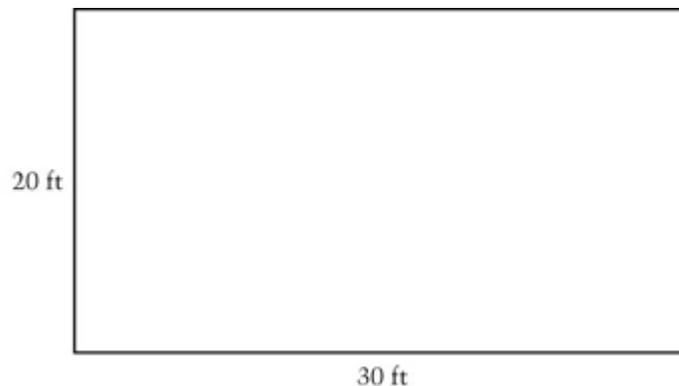
(b) Complete the table below by filling in the floor area, in square feet, for each room in your floor plan.

Room	Floor Area (in square feet)
Living	
Kitchen	
Bedroom	
Bedroom	
Bathroom	
Total Floor Area	600

TASK R:
Lucy wants to make a game spinner in which the chance of landing on green will be twice the chance of landing on yellow. She is going to label each section either green (G) or yellow (Y). Show how she could label her spinner. *(Adapted from NAEP, Grade 4, 2007, Released Questions)*
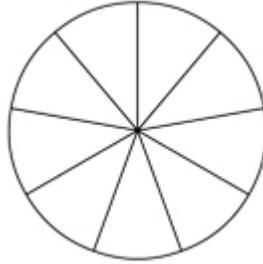
Number of Greens (G) _____   Number of Yellows (Y) _____

Explain how you found your answer.

LIST OF MATH TASK RESOURCES FOR TEACHERS

Books

Burrill, John. *Mathematics Assessment Sampler, Grades 6-8: Items Aligned with NCTM's Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2005. Print.

Schuster, Lainie, and Nancy Canavan. Anderson. *Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5-8*. Sausalito, CA: Math Solutions Publications, 2005. Print.

Erickson, Tim. *Get It Together: Math Problems for Groups, Grades 4-12*. Berkeley, Calif.: EQUALS, Lawrence Hall of Science, University of California, 1989. Print.

Erickson, Tim. *United We Solve: Math Problems for Groups, Grades 5-10*. Oakland, Calif.: Eeps Media, 1996. Print.

Wheatley, Grayson H., and George E. Abshire. *Developing Mathematical Fluency: Activities for Grades 5-8*. Tallahassee, FL: Mathematics Learning, 2002. Print.

Web Sites

Open-Ended Math Problems: http://sln.fi.edu/school/math2/

Problems with a Point: http://www2.edc.org/MathProblems/

Middle School Portal/Math Assessment:
http://wiki.nsdl.org/index.php/MiddleSchoolPortal/Math_Assessment#Problems_for_Practice

Problem-Solving for Middle School:
http://msteacher.org/epubs/math/math2/projects.aspx

NCDPI Middle Grades Resources:
http://www.dpi.state.nc.us/curriculum/mathematics/middlegrades/

Illuminations: http://illuminations.nctm.org/

NCTM Middle Grades: http://www.nctm.org/resources/middle.aspx

Journals
Mathematics Teaching in the Middle School