

Class Discussions

There are many benefits to high quality class discussions of mathematical ideas

- Opportunities for students to participate in mathematical thinking
- Chances for students to reflect on their own thinking processes
- Reasoning and logic are promoted as students question others, defend their position, and/or provide examples or counter-examples

-adapted from Chapin, et al, *Classroom Discussions: Using Math Talk to Help Students Learn*, Math Solutions Publications, 2003

Classroom discussions – at least those of high quality which are mathematically productive – have many benefits for promoting student thinking and learning. Discussions give students more opportunities to observe and listen to mathematical thinking in their peers as well as to participate in the conversation themselves.

“[In] whole-class discussion, the teacher ... is attempting to get students to share their thinking, explain the steps in their reasoning, and build on one another’s contributions.... The teacher facilitates and guides quite actively, but does not focus on providing answers directly. Instead, the focus is on the students’ thinking.”

-Chapin, et al, *Classroom Discussions: Using Math Talk to Help Students Learn*, 2003, p. 17

Guidelines to Evaluate Responses to Questions/Problems

1. What do I know about students’ understanding when they answer this question/problem?
2. Do students know the answer or how to get the answer as soon as they finish reading the problem? [Is it a real problem and not just an exercise?]
3. Is there more than one way to find the solution for the problem?
4. Can I ask more than one question about this problem?
5. Can students generalize from working with this problem?
6. Can I reverse the process and ask an interesting question?
7. What happens if...?

Asking Good Questions

Necessary traits for an interviewer

- ✓ Patience
 - “Children must be allowed time with their own minds.”
-Gwen Clay, Meredith University
 - When the interviewer and the student become comfortable with periods of silence, the quality of the interview is increased.
- ✓ Persistence
 - Keep the original emphasis for why you are asking questions or interviewing, but
 - Pursue worthwhile questions that come up in interview

Patience:

Time is always an issue for teachers, but when interviewing a child to get at his/her thinking it is necessary to take the time to let the child think about his/her thinking and work on putting that thinking into words. Put yourself in the child's place. We do not want to be asked a question, especially one that is probing our deeper understanding of an idea, and then be expected to give an answer within a few seconds. And we might want to be able to think on our feet and revise our thinking as we talk about it. Children should be given the same opportunity. It is also important for us as teachers to understand that silence is not necessarily a sign of a lack of understanding, but may be the opportunity for the child to organize his/her thoughts or to clarify them or to put them into words. A musician once said that the silences in a piece of music are as important and as much a part of the whole piece of music as the notes are. In the same way, the silences in a conversation or interview of a child are as important as the words to the final product of finding out about the child's real thinking and understanding.

Persistence:

It is easy to give up when a child does not give a quick response or a first response that is the one we want, but the job of the teacher/interviewer is to keep in mind what he/she is trying to find out, to think carefully about the next question based on what he/she is learning from the child, and to keep the questioning environment one that is safe enough and supportive enough for the child to give honest responses and to perhaps correct his/her own thinking.

Creating an Assessment

Clarifying Objective: Use strategies with 3-digit by 1-digit division with and without remainders to develop fluency.

Learning Target: Use a break-apart strategy to decompose a number with three digits.

Assessment:

1. Break 425 apart in one way.
2. Break 613 apart in two different ways.
3. Break 347 apart in three different ways.

Learning Target: Use a break-apart strategy to decompose a number in preparation for dividing by one digit.

Assessment:

1. Show one way to break apart 216 for this problem: $216 \div 3$
2. Show one way to break apart 216 for this problem: $216 \div 4$
3. Explain why you might need to break 216 apart differently for different divisors.

Learning Target: Use a break-apart strategy to divide by one digit.

Assessment:

1. Use a break-apart strategy to solve $248 \div 2$
2. Use a break-apart strategy to solve $359 \div 7$
3. Use a break-apart strategy to solve $783 \div 4$

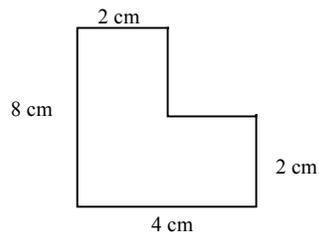
Changing Targets in Mid-Stream

A fifth grade teacher was starting a unit on area and perimeter. She knew that the students had an introduction to area and perimeter in fourth grade, so her plan was to build on that with some extension activities, e.g., finding areas of irregular shapes made of rectangular regions. Her learning targets included

- Find the area and perimeter of irregular shapes composed of rectangular regions
- Find the area of a rectangular region surrounding another rectangular region, e.g., the walkway around a garden or pool
- Explore the area of triangles as half of the area of a rectangular region
- Explore the relationship of area to perimeter and vice versa

She planned to start with this problem:

Find the area of this shape:

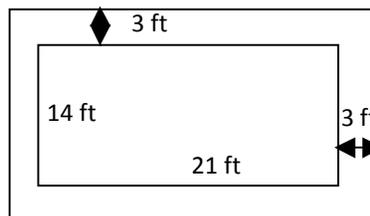


She decided to do a quick pre-assessment to make sure the students knew what she expected them to know. As the students wrote answers to her questions, she realized that there was not as much recall of the basic concepts of area and perimeter as she expected. A major gap was a lack of understanding of the concept of a unit. Some students didn't know or recall the meaning of perimeter. Some couldn't recall how to find the area and/or the perimeter.

She went ahead with her plan of giving the students the problem above, asking them to work with a partner or group to solve and discuss their solutions. Then she had them share the different ways that they solved the problem. This was a stretch for some students, but most seemed to have some refreshed understanding after doing this activity.

She had planned to continue the extensions of area work during the next class with this problem:

A garden is 14 feet by 21 feet. It is surrounded by a walkway that is 3 feet wide as shown. Find the area of just the walkway.



After giving the pre-assessment and seeing the students work on the first problem, she realized that her learning targets had to change immediately. She decided that it was necessary to review the fundamental concepts of area and perimeter, filling in gaps in either understanding or recall. Her learning targets expanded to include

- Understand the meaning of “unit” as used in measurement, and in area and perimeter in particular
- Know the units in the customary and metric systems used for measuring area and perimeter
- Be able to choose an appropriate unit for measuring a region to find the area and/or perimeter
- Measure a rectangular region and accurately find its area and perimeter

On the second day, with these new targets in mind, she chose to focus first on the customary measurement system. She had the students fill out a chart with the customary units used to measure

perimeter (inches, feet, yards, miles) and area (square inches, square feet, square yards, square miles), and find the equivalences among the units of area in particular. She used models of the units which they had created in the previous year to help them find these equivalences. The students kept this chart in their notebooks for future reference. She then had them work in pairs to 1) choose a region to measure in one of the customary units, 2) draw a representation of the region on dot paper in order to see the square units, and 3) find its area, showing the equation and labeling the area with the correct unit. A class discussion followed in which the pairs shared the region they chose, the unit they chose to measure in, and the area of their region.

In the next class, she chose to focus on the equivalencies between the units of area in the customary system. She had the class brainstorm some available regions that would make sense to measure in feet. In pairs, the students chose a region to measure, drew a representation of it (this time without the dot paper), and found the area in square feet. Using the chart made the day before and the area in square feet, the students found the area in square yard (having to divide the number of square feet by 9) and in square inches (having to multiply the number of square feet by 144). They then found the perimeter in feet, yards, and inches. On following days, she replicated these activities using the metric measurement units for perimeter and area.

Following these activities, she was able to go back to the walkway around the garden problem and have the students work in pairs to solve it. For those who finished before others, she asked them to find a second way to solve. After all students had the chance to at least give the problem some good thought (with some needing scaffolding questions from the teacher or peers), she debriefed the problem with the whole class, having students share their various ways of solving it.

After building this foundation for some of the fundamental ideas related to area and perimeter, she looked for (and created some) good problems that would give meaningful practice related to her original targets of finding areas of regions beyond a simple rectangle. She was able to plan differentiated activities, moving into some more exploratory activities with some students who showed a mastery level of understanding of these ideas, and providing meaningful but less complex activities for students who needed more experience with finding areas and perimeters. Allowing both groups to work as pairs or small groups, along with her interactions with individuals or small groups, provided the scaffolding each group needed to feel and be successful.

The moral of this true story is that as careful as we may be in planning learning targets for our units and individual lessons, what we learn from student work, student responses, and conversations with students, can and sometimes (maybe often) will cause us as teachers to change our targets. We may find, as in this case, that the students are not as prepared as we thought for tackling the targets we have in mind. We may likewise find that the students (or at least some students) have a deeper understanding or knowledge of the concepts we plan to present and our targets need to change to ones that delve more deeply into the mathematics. This is formative assessment in action – what we learn from working with our students informing our learning targets and instructional plans.

Vygotsky's Zone of Proximal Development

“Zone of Proximal Development” or ZPD as defined on wikipedia.com: “ZPD is Vygotsky’s term for the range of tasks that are too difficult for the child to master alone but that can be learned with guidance and assistance of adults or more-skilled children. The lower limit of ZPD is the level of skill reached by the child working independently. The upper limit is the level of additional responsibility the child can accept with the assistance of an able instructor. The ZPD captures the child’s cognitive skills that are in the process of maturing and can be accomplished only with the assistance of a more-skilled person. Scaffolding is a concept closely related to the idea of ZPD. Scaffolding is changing the level of support. Over the course of a teaching session, a more-skilled person adjusts the amount of guidance to fit the child’s current performance. Dialogue is an important tool of this process in the zone of proximal development. In a dialogue unsystematic, disorganized, and spontaneous concepts of a child are met with the more systematic, logical and rational concepts of the skilled helper.”

--From Santrock, J (2004). *A Topical Approach To Life-Span Development*. Chapter 6 Cognitive Development Approaches (200 – 225). New York, NY: McGraw-Hill, quoted on wikipedia.com

Zone of Proximal Development

- ✓ The best social learning occurs when conversations in the classroom are within the child’s Zone of Proximal Development
- ✓ Classroom discussion based on students’ own ideas and solutions to problems is absolutely “foundational to children’s learning.”
- ✓ “Scaffolding” refers to the way the adult guides the child’s learning via focused questions and positive interactions.

-Balaban, Nancy. (1995) "Seeing the Child, Knowing the Person." In Ayers, W. "To Become a Teacher," Teachers College Press. (quoted in http://en.wikipedia.org/wiki/Zone_of_proximal_development)

Differentiated Instruction

Principles of Differentiation

“There is no recipe for differentiation. Rather, it is a way of teaching and learning that values the individual and can be translated into classroom practice in many ways.”

-Carol Ann Tomlinson, University of Virginia

This quote from Carol Ann Tomlinson at the University of Virginia indicates that differentiation is not a program or formula, but is the result of a teacher’s mindset that each child is valuable and worthy of instruction from which he/she can learn, and takes many varied forms.

While there is no recipe, Tomlinson outlines principles which do apply to a differentiated classroom:

- 1) Assessment is ongoing and tightly linked to instruction – That’s what formative assessment is all about;
- 2) Teachers work hard to ensure “respectful activities” for all students – equally interesting, appealing, focused on essential understandings and skills, and accomplished through worthwhile meaningful tasks for all. This is an especially important point. The interventions for children who are struggling or “don’t get it” should not be more of the same that already hasn’t worked for them. It should be as interesting and engaging as the activities that the children who “got it” are doing; the difference is that the interventions for struggling children are geared to their needs as are the more complex or challenging activities given to the children who need those.
- 3) Students are allowed to work with a variety of their peers over the course of days, including whole group activities. There is no “tracking.” Groupings are flexible not rigid.

“Differentiated Instruction is an organized, yet flexible way of proactively adjusting teaching and learning to meet students where they are and help all students achieve maximum growth as learners.”

-Carol Ann Tomlinson (1999). *How to Differentiate Instruction in Mixed-ability Classrooms*. Alexandria, VA : ASCD.

Tomlinson makes the point that all instruction should be based on “best practices,” And that there is no point in differentiating instruction unless you’re beginning with instruction that ranks as best practice. "The starting point is what you need to do to challenge the highly able student. What you’re then doing is insuring that all kids get the best-practice instruction. Whenever you have teachers doing that, it reshapes how they teach all kids.“

(Carol Ann Tomlinson quoted in Mary Ann Hess, *Although Some Voice Doubts, Advocates Say Differentiated Instruction Can Raise the Bar for All Learners*, www.weac.org/Home/Parents_Community/differ.aspx

What is Differentiated Instruction?

Instruction may be differentiated in

- ✓ Content (what students need to learn)
- ✓ Process (how they will learn it: what children do to practice or make sense of the content)
- ✓ Product (how they express what they have learned: the outcome of the lesson or unit)

This list is what is being differentiated. In differentiated instruction the teacher proactively plans *varied approaches* to what students need to learn (*content*), how they will learn it (*process*), and/or how they can express what they have learned (*product*) in order to increase the likelihood that each student will learn as much as he or she can as efficiently as possible. (Tomlinson, 2003, p. 151)

The content is the input – the material being presented, the learning target for the lesson or unit. The process is what the children actually do to make sense of the content, to learn it in meaningful ways, or to practice it to solidify understanding and recall. The product is what the student uses to express the understandings and skills learned through the lesson or unit. It might be a test, a project, a report, a poster, or any other expression of what the child has learned. (From: “Tiered Lessons: One Way to Differentiate Mathematics Instruction” by Rebecca L. Pierce and Cheryl M. Adams in *Math Education for Gifted Students*, Prufrock Press)

Differentiation may be based on

- ✓ Readiness
- ✓ Interest
- ✓ Learning Profile

These are the ways in which instruction can be differentiated.

Readiness refers to prior knowledge and a student’s current skill and proficiency with the material presented in the lesson. It is largely readiness that will be determined by formative assessments.

Interest and *Learning Profile* must also be considered as lessons are planned so that children are *engaged* in activities through which they are able to learn depending on *how they learn best*. When determining what a student doesn’t know or understand or doesn’t “quite get,” we must consider the kinds of activities to which he/she has already been exposed in terms of his/her learning profile. Has the kinesthetic learner been able to use “hands-on” materials? Has the child who works best alone been able to do that? Has the auditory learner only been exposed to visual representations?

It is also important to note that content, in particular, can be differentiated in response to any combination of readiness, interest, and learning profile.

A Quick List of Methods for Differentiating Instruction:

- Flexible groupings
- Tiered Assignments
- Choices/Anchors
- Learning Contracts
- Compacting
- Mini-lessons
- Scaffolding

While there is no recipe for differentiation, there are proven methods, including these on this quick list. The choice of method will depend on the inferences made about what individual children or groups of children need. Some of these will be discussed more thoroughly in the next slides.

Flexible Grouping

Flexible grouping is a hallmark of a differentiated classroom.

Tomlinson: "We know huge amounts about how individuals learn. Most of us have memories of being in places where we thought we were going to scream if someone repeated one more time something we'd understood seemingly forever — and places where we were about to explode with frustration because we simply could not grasp the ideas at the pace they were presented. We also all know what a difference it makes if we can work alone when we need space to think things through by ourselves, or work in a group when we need sounding boards."

If we know and respect these things about ourselves, she asks, don't we owe the same to our students?

Flexible groupings allow teachers to work with students with similar needs, students to see themselves in a variety of contexts, and teachers to see students in different settings and with different kinds of work.

Groupings are FLEXIBLE – not rigid – and may take many forms, depending on what the need is for the current task, topic, concept, etc.

Tiered Assignments

From enhancedlearning.ca: "Tiered activities are a series of related tasks of varying complexity. All of these activities relate to essential understanding and key skills that students need to acquire. Teachers assign the activities as alternative ways of reaching the same goals taking into account individual student needs."

From Pierce and Adams, Prufrock Press

"Tomlinson (1999) described tiered lessons as 'the meat and potatoes of differentiated instruction.' A tiered lesson is a differentiation strategy that addresses a particular standard, key concept, and generalization, but allows several pathways for students to arrive at an understanding of these components based on their interests, readiness, or learning profiles."

Anchoring Activities

Anchors (or anchoring activities) may be a list of activities that a student can do to at any time when they have completed present assignments or it can be assigned for a short period at the beginning of each class as students organize themselves and prepare for work. These activities may relate to specific needs or enrichment opportunities, including problems to solve or journals to write. They could also be part of a long-term project that a student is working on. These activities may provide the teacher with time to provide specific help and small group instruction

to students requiring additional help to get started. Students can work at different paces but always have productive work they can do. Some time ago these activities may have been called seat-work, and should not be confused with busy-work. *These activities must be worthy of a student's time and appropriate to their learning needs.*

Learning Centers

Learning Centers have been used by teachers for a long time and may contain both differentiated and compulsory activities. However a learning centre is not necessarily differentiated unless the activities are varied by complexity taking in to account different student ability and readiness. It is important that students understand what is expected of them at the learning centre and are encouraged to manage their use of time. The degree of structure that is provided will vary according to student independent work habits. At the end of each week students should be able to account for their use of time.

Adjusting Questions

From enhancedlearning.ca: During large group discussion activities, teachers direct the higher level questions to the students who can handle them and adjust questions accordingly for student with greater needs. All students are answering important questions that require them to think but the questions are targeted towards the student's ability or readiness level.

Making Differentiation Work

From Mary Ann Hess, *Although Some Voice Doubts, Advocates Say Differentiated Instruction Can Raise the Bar for All Learners*, www.weac.org/Home/Parents_Community/differ.aspx

How? In classrooms where differentiation is alive and well, teachers:

- Keep the focus on concepts, emphasizing understanding and sense-making, not retention and regurgitation of fragmented facts.
- Use ongoing assessments of readiness and interests, and pre-assess to find students needing more support and those who can leap forward. They don't assume all students need a certain task.
- Make grouping flexible. They let students work alone sometimes and also in groups based on readiness, interests, or learning styles. They use whole-group instruction for introducing ideas, planning, or sharing results.
- See themselves as a guides. They help students set goals based on readiness, interests, and learning profiles — and assess based on growth and goal attainment.

See www.enhancelearning.ca for other differentiation information.

A Short List of Resources for Differentiation:

Tomlinson, Carol Ann. *How to Differentiate Instruction in Mixed Ability Classrooms (2nd Edition)*. 2 ed. Columbus, Ohio: Association For Supervision & Curriculum Development (ASCD), 2004.

Eidson, Caroline Cunningham, and Carol Ann Tomlinson. *Differentiation in Practice: A Resource Guide for Differentiating Curriculum, Grades K-5*. Alexandria, VA: Association For Supervision & Curriculum Development, 2003.

Eidson, Caroline, Robert Iseminger, and Chris Taibbi. *Demystifying Differentiation in Elementary School Book and CD*. Beavercreek, OH: Pieces Of Learning, 2008.

Dacey, Linda, and Jayne Bamford Lynch. *Math For All: Differentiating Instruction, Grades 3-5*. Math Solutions Publications, 2007.

Mctighe, Jay, and Carol Ann Tomlinson. *Integrating Differentiated Instruction & Understanding by Design (Connecting Content and Kids)*. Alexandria, VA: ASCD, 2006.

<http://www.caroltomlinson.com/index.html>

<http://www.ericdigests.org/2001-2/elementary.html> (Eric digest article – full text)

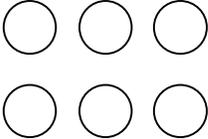
http://www.prufrock.com/client/client_pages/GCT_Readers/Math/Ch._4/Tiered_Lessons_for_Gifted_Children.cfm (PDF of brief above)

<http://www.ehancelearning.ca> (click on links to differentiation information)

<http://members.shaw.ca/priscillatheroux/differentiatingstrategies.html>

Scaffolding with Questions: One Scenario

A fourth grade child was given a task involving shading fractional parts of sets. She was having difficulty even knowing how to begin. The teacher sat with her and asked questions to scaffold her understanding of the task and of the mathematics underlying the task. Here is the conversation.

The task: Shade $\frac{2}{3}$ of this set: 

The conversation:

Teacher says	Hoped for response from student	Student says
What is this task asking you to do?	Shade $\frac{2}{3}$ of the circles	Shade $\frac{2}{3}$.
What does that mean that you do?	Divide the circles into 3 groups and color 2 of the groups	Divide the circles into groups
What do you already know about that will help you do that?	The denominator tells how many groups and the numerator tells how many to shade	The bottom number tells how many groups (Then circles three groups of 2)
Tell me about your groups.	There are three groups and each has 2 circles. They are equal groups.	I made 3 groups and each group has 2.
How can you know what to do next?	The numerator tells how many of the groups of 2 to shade.	Shade two because the top number is a two. (Then colors two circles in one group.)
What have you colored?	I shaded $\frac{2}{3}$ of the set. (Then realizes that she has shaded only two circles, not two groups.)	I shaded 2 like the top number.
Show me how what you shaded represents $\frac{2}{3}$ of the set.	Oops! I shaded only one of the three groups. I should have shaded two groups.	Oh, I only shaded one group. I need to shade another one. (Then shades another group of 2 circles.)
Now tell me how you know that you have shaded $\frac{2}{3}$ of the set.	I made three equal groups and colored in two of the groups – 4 circles in all.	I've colored two of the three groups I circled.
How many circles are in $\frac{2}{3}$ of this set?	Four – two groups of two	Four.

Graphic Organizers help scaffold learning

A.

Ready-made chart with or without a word bank (with or without the words provided – student could add his/her own words)

Example:

How are a square and a rectangle alike and different?		
Alike	Different	<div style="border: 1px solid black; padding: 5px; min-height: 100px;">Word Bank</div>

B.

Structured organizer to help student work through a problem

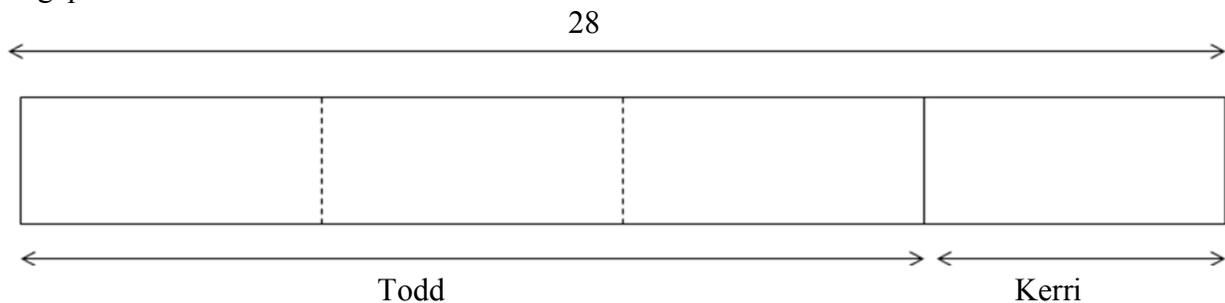
Example:

Together Todd and Kerri earned 28 points in the basketball game. Todd earned 3 times as many points as Kerri. How many points did Todd earn?

FACTS (or “What do I know?”)	DRAWINGS (or “What do I need to find out?”)
COMPUTATION (or “My Work?”)	SOLUTION

OR

Singapore Math “bar” model



Graphic Organizers help scaffold learning

C.

Space provided for work and solution

Example:

<p>Tim had \$1.00 in coins. He had 15 coins which were only dimes and nickels. How many of each kind of coin did he have? <i>Work Space:</i></p> <p>Solution: _____dimes _____nickels</p>

D.

Presenting the same type of situation in different ways

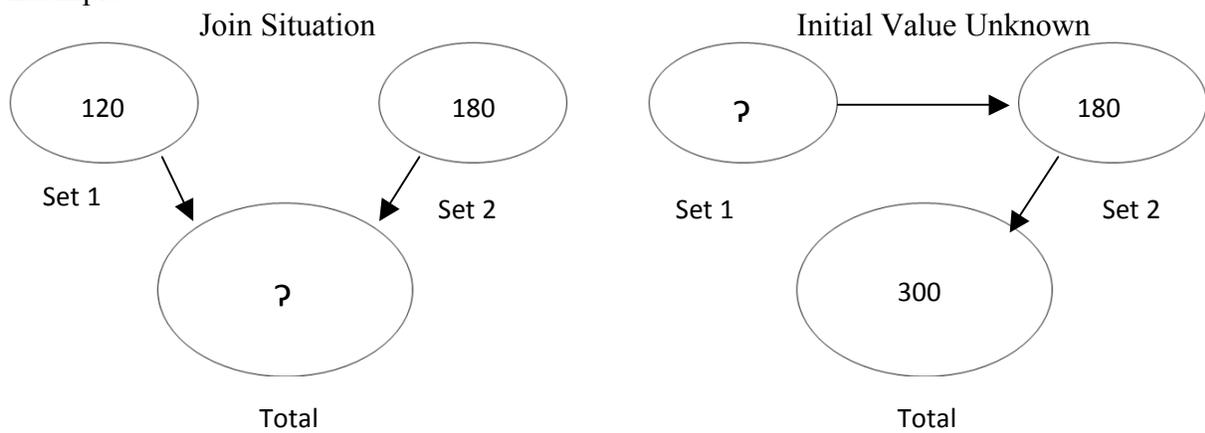
Example:

- Sean drove 120 miles and stopped for lunch. Then he drove another 180 miles before he reached his destination. How many miles did Sean drive?
- Sean drove 120 miles and stopped for lunch. Then he drove some more. By the time he got to his destination, he had driven 300 miles. How many miles did he drive after lunch?
- Sean drove for awhile before he stopped for lunch. After lunch he drove 180 miles to reach his destination. When he got there he had driven 300 miles. How many miles did Sean drive before lunch?

E.

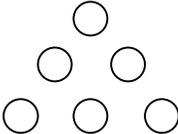
Models of Part – Part –Whole

Examples:



Arrows may or may not be included to show the direction of change.

Choose and complete one activity in each row.

<p>Draw a picture that shows a model of $365 - 248$. Think about how your drawing connects to how you use paper and pencil to find the difference. Discuss your ideas with a friend.</p>	<p>Your friend subtracted 408 from 963 and got 565. How could you show your friend that her answer is wrong? What would you tell her?</p>	<p>Write directions for two different ways to find the difference between 700 and 365 when you solve by using paper and pencil.</p>						
<p>Use the numbers 2, 4, 5, 10, 25, and 20 to place into the diagram so that the product of each side is 200.</p> <div style="text-align: center;">  </div> <p>Write one more problem like this one and trade it with a classmate.</p>	<p>Place a subtraction sign to make an equation that is true.</p> $1084688 = 396$ <p>Create two more problems like this one and trade them with a classmate. (You solve his or hers and he/she solves yours.)</p>	<p>Which two numbers should you exchange so that the three numbers on each card can be used to create a true subtraction equation?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">343</td> <td style="padding: 5px;">642</td> <td style="padding: 5px;">406</td> </tr> <tr> <td style="padding: 5px;">797</td> <td style="padding: 5px;">1000</td> <td style="padding: 5px;">1203</td> </tr> </table> <p>Create two more problems like this one and trade them with a classmate.</p>	343	642	406	797	1000	1203
343	642	406						
797	1000	1203						
<p>Make a list of ways you use subtraction outside of school.</p>	<p>Interview a classmate or other friend about what he or she knows about subtraction. Find out as much as you can in three minutes. Write a report giving suggestions for teaching about subtraction.</p>	<p>A story problem can be solved by adding 42 and 16 and then subtracting 12. Write two interesting story problems that could have been solved that way.</p>						

Row 1: Focus is on communication of ways to find differences.

Row 2: Provide practice with subtraction and the guess-and-check method of problem solving

Row 3: Goal is to make connections to a) the real world, b) a peer's way of thinking about subtraction, and c) story problems.

Students may be asked to complete one task in each row.

A similar Think-Tac-Toe could be organized so that students would be asked to complete all three tasks in a given row (or a choice of rows). Choices could be tiered, with each row containing more challenging tasks or tasks geared to different learning styles.

Adapted from example in Dacey and Lynch, *Math for all: Differentiating Instruction*, Math Solutions Publications, 2007.

Think-Tac-Toe Number and Operation: Multiplication and Division Grade 4

Choose and complete one activity in each row.

<p>The answer to a division question is 4. What might the question be? Is there more than one possibility? What strategies did you use?</p>	<p>When the children in Mrs. Short's fourth grade class got into groups of three, one child was left over. How many children might have been in Mrs. Short's class? Is there more than one possibility? Write about your thinking as you answered this question.</p>	<p>Twenty-four people in the Folk Dance Club were learning a dance that had to be done in equal size groups. How many different ways could they be grouped? What if the groups did not have to be equal in size? What different ways could the dancers be grouped then if no one could be left out? Show your results to each question clearly.</p>
<p>What could you add to 461 to make it divisible by 10? How many possibilities can you find? Can you find a rule to help you?</p>	<p>Two numbers are multiplied to give 36,000. What might the two numbers be? How many different answers can you find? Write about your thinking as you looked for possibilities.</p>	<p>My school has 500 students. Each child comes to school on the bus and every bus carries the same number of students. How many students might be on each bus? Show your results in an interesting way.</p>
<p>Find two numbers that multiply to 2,280. How many different answers can you find? What was your strategy?</p>	<p>Use five 4's and any operation to make the whole numbers from 0 to 50. You may do this with a friend. Make a chart of your results. Be sure to use order of operations as you write your results. Write a journal entry about the strategies you used. What other numbers can you make using five 4's?</p>	<p>Use all of the single digits except zero (1, 2, 3, 4, 5, 6, 7, 8, and 9) and any operations. What is the smallest possible number you can make? What is the largest possible number you can make? How do you know that you have found the largest and the smallest? What other numbers can you make? Make a poster to show your results.</p>

Choose and complete one activity in each row.

<p>Make five different squares on a geoboard. Record your squares on geopaper. Compare your squares with a classmate's squares. Are any of them the same? Which ones are different?</p>	<p>You want your friend to draw a trapezoid, but you cannot tell him that it is a trapezoid you want him to draw. Write a description of what you want him to draw without using the word "trapezoid." Try to follow the description yourself to see if it works. Then get a friend to try. If your friend can't draw a trapezoid from your description, figure out how to improve your description. Turn in both your original description and the new one if you had to revise it.</p>	<p>Draw or cut out a variety of quadrilaterals, including squares, rectangles, parallelograms, and trapezoids. You may draw others as well. Draw in the lines of symmetry for each shape. Make a poster of your results. Be sure you have clearly shown the lines of symmetry in each shape. What do your results tell you about quadrilaterals and line symmetry?</p>
<p>Using a geoboard, find out how many squares of any size that you can make on the geoboard. Record your findings on geopaper. Is there a pattern that would help you find them all and/or count them? Describe the pattern.</p>	<p>Your friend drew a shape and said that it was a rectangle. You don't think it really is a rectangle. What things would you do to try to prove to your friend that her shape is not a rectangle? What would you be looking for? Write a letter to your friend to tell her why her shape is or is not a rectangle.</p>	<p>Draw or cut out a variety of quadrilaterals, including squares, rectangles, parallelograms, and trapezoids. You may draw others as well. Draw in the lines of symmetry for each shape. Determine the number of turns of rotational symmetry each shape has, if any. Make a poster of your results. Be sure you have clearly shown the lines of symmetry and the rotations of symmetry.</p>
<p>Create a shape with overlapping squares and rectangles. Find out how many total rectangles are in your design. Make several copies of your design and have at least 3 other people try to find all the rectangles. What happened? Did they find them all? Did they find ones that you didn't see? If they said that yours was easy, try to make a harder one. If they thought yours was hard, try to make an easier one. Turn in both.</p>	<p>Go on a scavenger hunt for quadrilaterals in the world around you. Take or draw pictures of them. Make a booklet of the quadrilaterals you find. Put them in groups by the kind of quadrilateral. Label your pictures and tell where you found the shape. Which different kinds of quadrilaterals did you find? Which ones were hardest to find? Were there any you couldn't find? Why do you think that is? Add the answers to these questions to your booklet.</p>	<p>A shape has at least two sides that are 5 cm long and at least two sides that are 10 cm long. One angle is a right angle, one angle is an acute angle, and the rest are greater than a right angle. Create some examples of what the shape might look like. Make a poster of your examples. Label the angles to show which ones fit the descriptions and label the lengths of the sides in centimeters. Find out the correct names of your polygons. Could any be quadrilaterals? Why or why not?</p>

FRACTIONS RAFT

ROLE	AUDIENCE	FORMAT	TOPIC
A box of 20 cookies	Person who bought the cookies	Drawings of cookies	Four different ways that I could be divided equally among some children
Numerator	Denominator	A "thinking of you" card	It's great to be your friend because both of us are needed to name a fraction
The fraction $\frac{1}{3}$	The fraction $\frac{1}{6}$	Poster	Why I'm a larger amount than you are
Student	Another student	Pattern blocks	Look at all the ways I can represent fractions using this set of blocks
The fraction $\frac{1}{5}$	Advice columnist	Letter asking for help	Why can't I ever be a part of the whole number family?
The fraction $\frac{1}{2}$	Audience for Awards Show	Top Ten List	The top ten fractions that name the same amount that I am
Fractions	Citizens of Your Town	Brochure	Ways your lives would be different if we didn't exist

Adapted from ideas in Eidson, Iseminger, and Taibbi. *Demystifying Differentiation in Elementary School: Tools, Strategies, & Activities to Use Now*, Pieces of Learning, 2008

TIME RAFT

ROLE	AUDIENCE	FORMAT	TOPIC
Teacher	Second Graders	Booklet	What you need to know about telling time
Marketing Company Employee	Consumers	Advertisement	Why analog clocks are the best
Committee of Teachers	Principal	Chart	Comparison of Schedules of Second and Fifth Grade Classes
Historian	Fourth Grade Students	Time Line	Important events in North Carolina history in the 1900s
Fourth Grade Students	Teacher	Report	Schedule of activities for the school's field day
Self	Parent	Schedule	The amount of time needed for after school activities each afternoon
Fill in your	idea here.	Check with teacher	for approval.

Adapted from ideas in *Dacey and Lynch, Math for All: Differentiating Instruction, Grades 3-5*, Math Solutions Publications, 2008