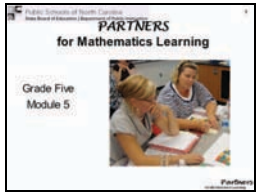
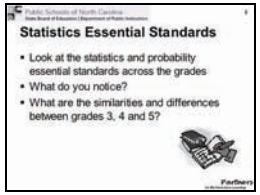
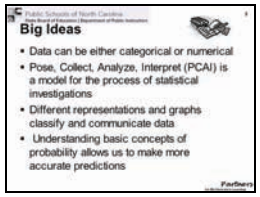
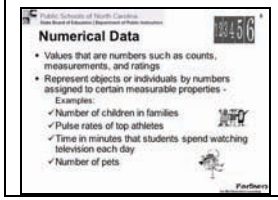
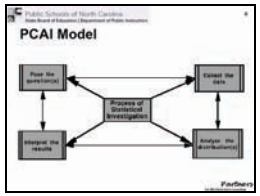
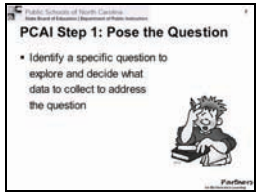
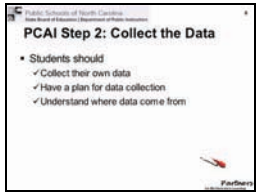
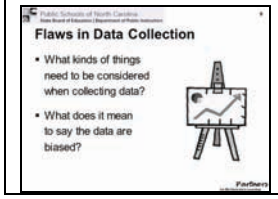
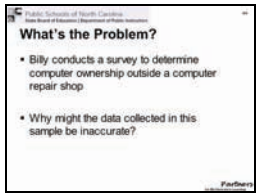
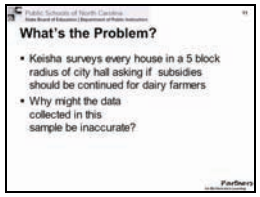
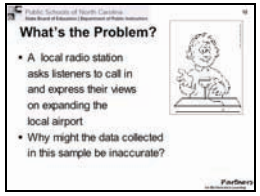
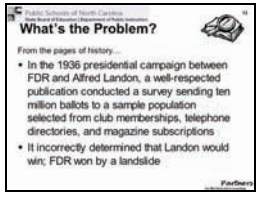
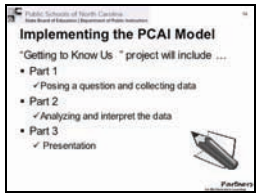
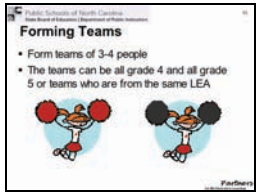
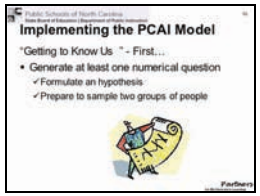
General Materials and Supplies:
Paper Meter sticks Base-10 blocks (2 flats, 20 rods, 20 cubes per pair of participants)
Calculator with fraction capability (e.g., Math Explorer) – optional Index card (1 per person)
Decimal/Fraction cards cut out (one set per table)

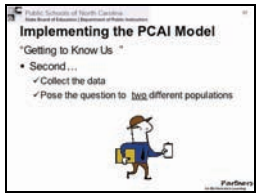
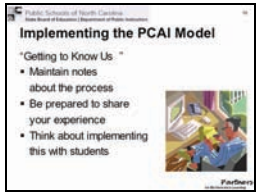
Slide	Tasks/Activity	Personal Notes
	(Slide 1) **Grade 5 Module 5** Statistics/Number and Operations; Greet participants. Then tell them to think about the process standards as you go through this module and to look for problem solving, reasoning and proof, connections to other strands, representations, and opportunities for communication. Explain that a single PowerPoint and set of leader notes will be used for both the fourth and fifth grade data modules. Participants in both grades will conduct a statistical investigation using the PCAI model that was introduced last year. However, some parts of the module are differentiated by grade level expectations. (Handouts are labeled by grade for example). As we go through the module, they should note places they need to make small adjustments if they are leading grade level sessions rather than the 4-5 band. Next give them the following "big picture" of what will occur during the statistics strand: • Review the big ideas introduced last year • Then the major focus of the module will be on the PCAI model for statistical investigations. We will conduct an investigation in which we go through all the steps of the model. In order to develop in depth understanding it is critical that children experience each phase of this process. • Participants will share their findings with the group. This module is purposefully short so that more time can be used in the number strand. Additionally, throughout all the modules there is deliberate integration of content. Measurement, for example, has several connections to data. Explain that the module has been divided into 2 parts to allow time for data collection and that other modules may come between part 1 and part 2 of data. The same PowerPoint will be used for both grade 4 and grade 5. However, the handouts are grade level specific.	

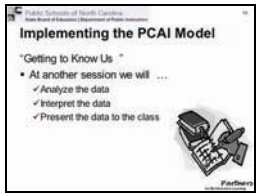
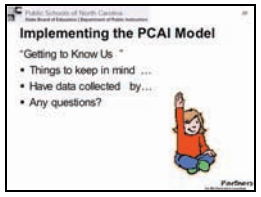
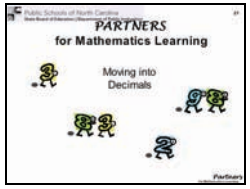
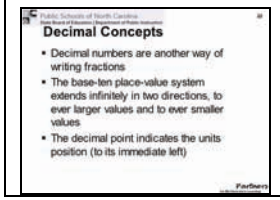
	(Slide 2) **Statistics Essential Standards** Direct participants to examine the data (or statistics and probability) strand K-5. They will note across the grades is the use of the process of statistical investigation. Ask them to quickly look at grades 3-5. What do they notice? Examples: In all three grades students conduct investigations but grade 3 displays data on a bar graph and in tables, grade 4 uses circle and line graphs, and grade 5 has stem and leaf plots (may be modified). Grade 3 introduces range, grade 4 adds mode and median, etc.	
	(Slide 3) **Big Ideas** Quickly review these. Tell participants our focus will be bullets 1 through 3 during this module.	
	(Slide 4) **Categorical Data** Remind participants that students need to distinguish between questions that give categorical and those that give numerical data before they pose a question and try to interpret the data. Ask participants what they recall about categorical data. Note that categorical data most often represent attributes like color, favorite food or car, which are not numerical. They are most often words. However categories can also be labeled with numbers like area code or zip code. Bar graphs are often used to represent categorical data. Ask if range can be found for categorical data? (No) Ask which of the measures of center (mean, median, mode) can be found for categorical data? (Only mode)	
	(Slide 5) **Numerical Data** Numerical data are often counts, measurements, and ratings. Note that unlike categorical data, measures of center and range can be found for numerical data. Ask which of the two kinds of data, categorical or numerical, children tend to do in the primary grades (categorical). Stress the importance of children collecting both kinds of data.	

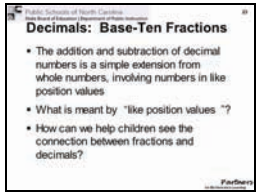
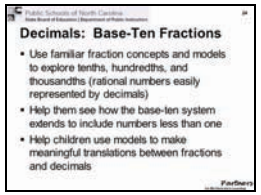
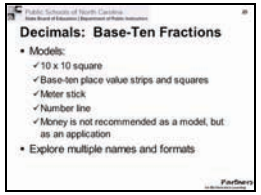
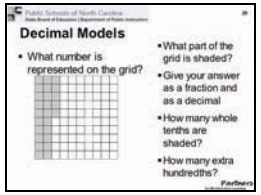
	BEFORE CLICKING TO NEXT SLIDE, ask participants recall the four steps in the PCAI model at their table.	
	(Slide 6) **PCAI Model** Show the slide for Big Idea number 2: the PCAI model for statistical investigation. Review the 4 components in the screen. Note the graphic illustrates that this process is not necessarily linear. Sometimes, investigators need to go back and revisit a previous part. The graphic shows the interrelatedness of the process.	
	(Slide 7) **PCAI Model Step 1:Pose the Question** Briefly review that the first step is to pose a question in order to hypothesize possible outcomes and make decisions about data collection. The question permits investigators to describe, summarize, compare/contrast information as well as identifying trends and making generalizations. Ask why this is important for children to do, rather than the teacher devising the question (taps at interests, engages learners, real life etc.). Designing the question can be challenging. Remind participants of questions that children may pose…"What is your favorite movie?" or What is your favorite flavor of ice cream?" may be student' first attempts. After collecting data with these kinds of questions, they will probably want to revise their question. While we can tell them to write "good" questions, experience will allow them to learn this.	
	(Slide 8) **PCAI Model Step 2: Collect the Data** Present the information on the slide. Note that children need multiple experiences with posing questions, refining them and collecting their own data. They need to decide on an organized method to collect and record data, making sure that people are not surveyed twice, the question isn't changed, etc. Often they are presented with data that have been collected and represented and then asked to interpret it. This does not help them understand where data come from.	
	(Slide 9) **Flaws in Data Collection** Briefly ask participants to respond to the question on the screen. When designing the data collection, care must be given when deciding on who is sampled and the size the sample as well as the wording used if questions are asked. Who to sample needs thought because data can be manipulated by controlling the sample group. For example, asking how much homework is appropriate would, in all likelihood, yield different results if asked to teachers as opposed to	

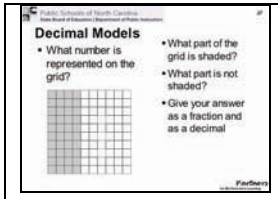
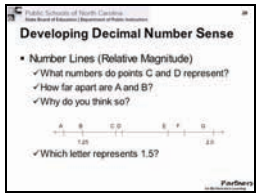
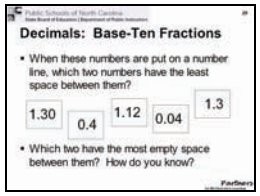
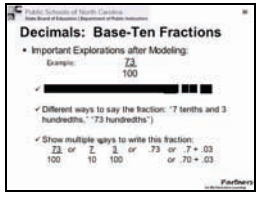
	fourth graders. However asking representatives from both groups for comparison purposes is fine. Ask participants to brainstorm some different ways data can be collected and some possible bias that could occur (Surveys, polls, observations, measurements, interviews, experiments, internet/media searches), meaning the data do not accurately represent the population in question. Then go to the next slides for some situations.	
	(Slide 10) **What's the Problem?** Have participants read the situation and discuss. Those frequenting the building do not represent the general population. They would be more likely to have a computer.	
	(Slide 11) **What's the Problem?** Read and discuss. The data would be skewed because only city dwellers would be polled. Farmers would not be included in the sample.	
	(Slide 12) **What's the Problem?** In this situation, usually only those with strong feelings would take the time to respond. It also includes only listeners to this particular show.	
	(Slide 13) **What's the Problem? From the Pages of History…** What might the data collected in this sample be inaccurate? The sample left out a large part of the population—the poor and uneducated—since this was during the Depression when phones, magazines and club memberships were luxuries. Tell the participants that experiences with actually collecting data is valuable to help them understand that data can be skewed because of situations like those mentioned on the previous	

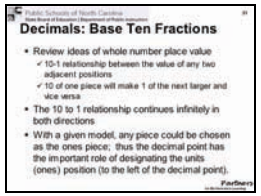
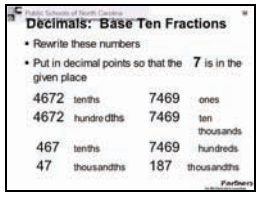
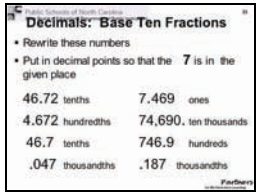
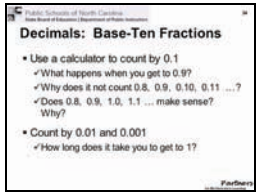
	slides. Care is needed in deciding who to survey and how many people to include in the sample. Ask participant groups to create other samples of survey bias and/or share examples of survey bias they have noticed/experienced. Note how the media frequently uses statements such as," Four out of five dentists prefer Crest," but we don't know how many dentists were surveyed or if those surveyed are employed by Crest.	
	(Slide 14) **Implementing the PCAI Model: Getting to Know Us** Explain we will go through the PCAI process as a part of this professional development. The big picture is that participants will be working today in a group to write a numerical question and make a plan for data collection. Then they will collect their data, and return on the next meeting date with their data. At that time we will continue our work with data and present it to the group. When doing this with teachers, you will need to plan how you will do this over time. Thus you'll need to know the schedule for the sixth module in order to allow some time to pass before coming back to this.	
	(Slide 15) **Forming Teams** For the summer training - ask participants to divide themselves somewhat evenly into 2 large groups; one will function as grade 4 and another will function as grade 5. Once they have formed 2 large groups (grade 4 and grade 5), direct them to form teams of 2-4 within their chosen grade group, forming small teams of participants to work on each grade level. Else, if participants are all 4th grade teachers have them form groups of 3 to 4 people. Tell participants that it would be ideal for these groups to be able to communicate outside of the workshop, but not necessary. The group will have the opportunity to devise a data collection plan that can be implemented by each individual.	
	(Slide 16) **Implementing the PCAI Model** Tell participants that today all groups will decide on a question and make plans to sample 2 groups of people. Then over the next few days (between workshop days) they will collect data and then return to complete the PCAI process in class. Explain that the overall purpose is to learn more about our group while practicing the PCAI	

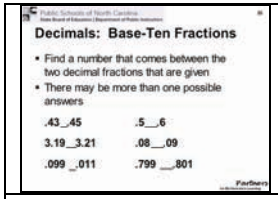
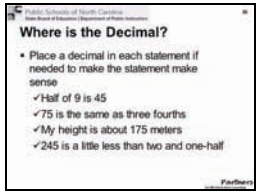
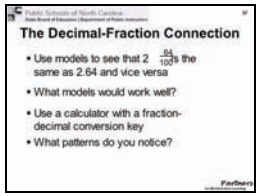
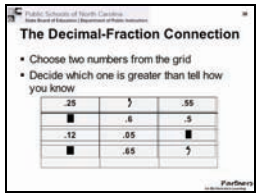
	model. Following that purpose, assume one of the groups sampled will be colleagues in this workshop, the other sample group will have to be chosen by each team. Provide the following guidelines for what they will be doing to class today: Formulate a numerical question and hypothesis. The question needs to be numerical to address other data objectives. However, they can pose a second question that is categorical if desired.) Depending on the group, leaders may want to have the participants brainstorm a list of possible questions (number of children, number of years taught, number of rings, number of tattoos, number of hours slept last night, number of miles traveled to get to this site, height, etc). Monitor to make certain groups are collecting different data.	
	(Slide 17) **Implementing the PCAI Model** Explain that in their small groups, they will make plans for how to collect data. They should keep in mind that they may need to revise their question. Ask what kinds of things might they have to consider when planning data collection. (How not to ask the same person more than once, how to organize their data so the 2 sample groups are not mixed, etc.) Have participants brainstorm a list of whom they might sample in addition to this class. Note: this is not the time to collect data from the group. Teams must have a plan for collecting data outside of workshop time.	
	(Slide 18) **Implementing the PCAI Model** Explain that during their presentation they will share reflections, problems, successes, rationale for choices, changes they would recommend, etc. For example, did they have to revise their question—how was it changed? Why? What problems occurred while collecting data? Therefore they must maintain notes/journal. Point out that students are to also keep notes/journal of their process and talk about their problems in their presentation**. We want the students to grapple with the problems they encounter**. They will become problem-solvers by encountering problems and finding solutions, revising their process. The teachers are not going through this activity so they can "fix" it for their students, but to experience the kinds of things there students may encounter. Tell participants to think about how they may want to represent their data but do NOT make a	

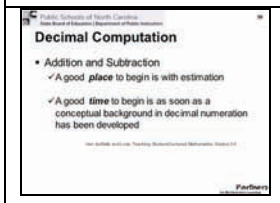
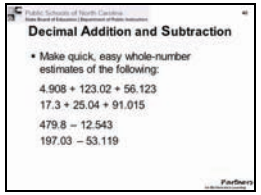
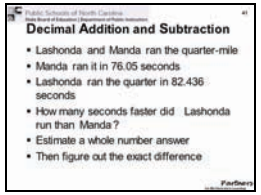
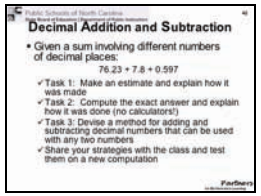
	display. We will do that in class.	
	(Slide 19) **Implementing the PCAI Model** Then at another session they will work to analyze, interpret the data and make a presentation of their work.	
	(Slide 20) **Implementing the PCAI Model** Brainstorm tips such as being sure to ask the question exactly the same way/decisions, how to record data collected, etc. Address any questions. Direct participants to look at handout titled Collection Plan. Ask them to use it during this part of their work. Groups then write their question. As they write, circulate to make sure the questions are numerical and that there are no repeats of questions. Attend to their plans for data collection. Make sure they have a way to collect the data that excludes collecting data from the same person twice and that they have an organized way to record the data. After groups have finished, you may have groups share their question if time allows.	
	(Slide 21) **Partners – Moving into Decimals** The second part of this module completes the number and operations strand with a focus on decimals. Remind participants to look for evidence of the process standards in the activities.	
	(Slide 22) **Decimal Concepts** These points are important ideas about decimals.	

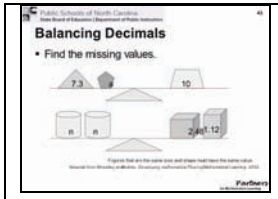
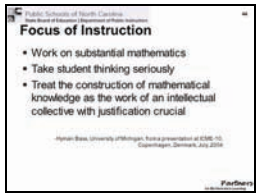
	(Slide 23) De**cimals: Base-Ten Fractions** Decimals build on the base 10 system of our whole numbers, so computing with decimals can build on the place value understanding built previously with whole numbers. Have participants suggest ways to help students make connections. Some ideas are offered on the next slide.	
	(Slide 24) **Decimals: Base-Ten Fractions** Bring out any points not made by the participants from the previous slide.	
	(Slide 25) **Decimals: Base-Ten Fractions** It is important to use a variety of models and types of models when working with decimal concepts.	
	(Slide 26) **Decimal Models** The 10x 10 grid as a model can help children see the relationship between tenths and hundredths. The shaded part is ¼ of the grid or 25/100 or .25. It can also be seen as 2/10 + 5/100 or .2 + .05. The unshaded part is ¾ of the grid or 75/100 or .75. All of these numerical representations are important for children to understand. (See handout " 10x10 grids".)	
	(Slide 27) **Decimal Models** The 10 x 10 grid can also be used to see the fractional parts that make up a whole, in this case 4/10 or .4 and 6/10 or .6 make up the total grid. Likewise, the equivalence of .4 and 2/5 can also be shown.	

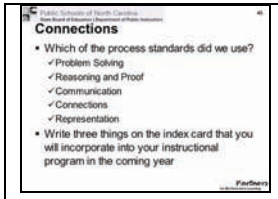
	Have the participants use the 10 x 10 grids in handouts to represent the following decimal fractions. For each have them write the various numerical representations shown: 35/100 = .35 = 3/10 + 5/100 = .3 + .05 42/100 = .42 = 4/10 + 5/100 = .4 + .02 89/100 = .89 = 8/10 + 9/100 = .8 + .09 6/10 = 60/100 = .6 = .60 It is important for children to see these kinds of relationships between the tenths and hundredths in any given decimal number.	
	(Slide 28) **Developing Decimal Number Sense** Number lines can be used to help children develop a sense of the relative magnitude of decimal numbers as well as whole numbers. Using this number line, ask the questions on the slide. Note that developing place value understanding and developing number sense with decimal numbers is not the same things. Children need activities that lead them to develop understanding of both.	
	(Slide 29) **Decimals: Base-Ten Fractions** This is a good assessment question. Have the participants put these numbers in order, then answer the questions. 1.3 and 1.30 are the closest because they are equivalent and represent the same spot on the number line. 0.4 and 1.12 would have the most space between them because there is more than .7 between them and no more than slightly less than .4 between any other two numbers (0.4 and 0.04 have 0.36 between them).	
	(Slide 30) **Decimals: Base-Ten Fractions** Talk about this slide. Ask participants what models they would use to explore this fraction. Have various models available for participants to use. Emphasize the many different ways that the amount 73/100 can be named. (Note another way to write 73/100 is 30/100 + 3/100.)	

	(Slide 31) **Decimals: Base-Ten Fractions** Decimals are an extension of the place value system. Children need a solid understanding of whole number place value before they begin to understand decimal notation. It is important that children recognize the ones place as the unit on which the other places are built.	
	(Slide 32) **Decimals: Base-Ten Fractions** This activity provides a good assessment of a child's understanding of decimal place value. See handout "What's the Number?" for more of this kind of activity.	
	(Slide 33) **Decimals: Base-Ten Fractions** This slide shows the solutions to the previous slide. Note where zeros had to be added to make the appropriate number.	
	(Slide 34) **Decimals: Base-Ten Fractions** The calculator is a good tool for helping children see patterns in consecutive numbers. Ask the questions under bullet one. Do children know why the count is not .9, .10, .11? What models and experiences will help them understand that? If you have time, have the participants begin to count by 0.01 (and 0.001) to 1 on the calculator to see how long it would take.	
	(Slide 35) **Decimals: Base-Ten Fractions** Have participants suggest numbers that are between each pair of decimal fractions. Discuss strategies. How would a number line help children do this kind of activity? A similar activity is on handout, "Find the Fraction".	

		
	(Slide 36) **Where is the Decimal?** A part of number sense is knowing when numbers make sense in a context. Statements like these in which children place a decimal to make them sensible help children relate their place value understandings to real life situations. See handout "Place the Decimal" for an activity like this one.	
	(Slide 37) **The Decimal-Fraction Connection** 10x10 squares or meter sticks are good models for point one. If you have the appropriate calculator, have participants convert fractions to decimals and vice versa. What patterns emerge? How would you choose the fractions so that students see patterns? After children have developed some number sense for fraction/decimal relationships, they can practice by using the cards in handout, "Decimal and Fraction Cards" to play Decimal War or Decimal Concentration. Another game that reinforces decimal place value and some decimal/fraction connections is Race to a Meter or Race to Zero. See handout, "Race to a Meter: A Decimal Game" in handout.	
	(Slide 38) **The Decimal-Fraction Connection** The "tell how you know" direction is especially important. The response will help you understand what your children understand about the relationship between different decimals, different fractions, and fractions and decimals. Have children write their responses to the prompt. Then have them choose a different pair of numbers from the grid, decide which one is *smaller* and tell why. Suggest that they create similar grids, including number words as well as the symbols, e.g., four hundred thousand or five tenths. Note that .65 and 2/3 are close. Children could check with a calculator by dividing 2 by 3 to	

	compare the decimal equivalent with .65. See handout "Bigger? Smaller? How do you know?" for similar charts.	
	(Slide 39) **Decimal Computation** Instruction in decimal addition and subtraction can start any time after the decimal number sense has been developed, but it is important to start with estimation not rules.	
	(Slide 40) **Decimal Addition and Subtraction** Estimates may be something like this: Addition #1: between 175 and 200 Addition #2: around 130 Subtraction #1: about 450, slightly more Subtraction #2: about 150, a little less	
	(Slide 41) **Decimal Addition and Subtraction** Have participants try this pretending they don't know the "line up the decimal" algorithm. They might note that 76.05 + 6 is 82.05, and then figure out how much more is needed to get to 82.436. They might count on from 76.05 by adding 0.95 to get to 82 seconds and then add on the remaining 0.436 seconds. Students will eventually confront the difference in the two-place decimal and the three-place decimal and begin to use their understanding of place value to resolve the issue.	
	(Slide 42) **Decimal Addition and Subtraction** After several chances to solve addition and subtraction story problems, this kind of activity can help children develop a method that works in any situation. It is reasonable to expect them to develop a method that is essentially the algorithm aligning the decimal points.	
	(Slide 43) **Balancing Decimals** The balance format brings algebraic thinking into the work with number, and helps to develop	

	meaning for decimals and operations with decimals. When using this format in working with decimals, students are more likely to develop solution strategies that are meaningful to them, rather than following a formula like "line up the decimal places." In the top example, participants must think "what added to 7.3 results in 10?" One strategy is to add .7 to 7.3 to get to 8.0, and then add another 2 to get to 10, the value of the trapezoid, so that a total of 2.7 was added and must be the value of the pentagon. In the second example, students would add 1.12 and 2.48 to get 3.60 or 3.6. Then they would have to find half of 3.6 (1.8). An extension would be to have participants come up with different values for the cylinder and prism if the value of the cube "n" stays the same. Before leaving this slide, be sure that participants see that activities like this involve both algebra and number/operations so that more than one strand can be addressed at the same time.	
	(Slide 44) **Focus of Instruction** These points were made by Hyman Bas, at the International Congress of Mathematical Educators in 2004. These ideas should be the focus of our instruction. The mathematics we are doing should be important "real" mathematics, not just fun activities or activities we have done with students for many years or just because it is in the textbook. It should match our Essential Standards, and also be meaningful to children by being problem based with applications in their real world. We as teachers must ask children to explain their thinking and then listen carefully to what they say. We need to know the mathematics enough that we can understand whether or not a child's thinking is mathematically accurate and understand the mathematics well enough to come up with strategies for helping children move their thinking to the next level. The work in the classroom should be that of a learning community (the "intellectual collective") where ideas are presented and explained by all members of that community, the classroom. Teachers can be learners too!	
	(Slide 45) **Connections** Have participants reflect on the activities in this module and share the problem solving that was done. What other process standards were utilized in this module? Participants should see places where	

	they used reasoning and proof (justification); where they communicated ideas, questions, reasoning; when they made connections between strands of math or connections to prior knowledge; and where they used representations to model a situation, to help understand a problem, or to write an equation or other representation in order to solve a problem. Help them see that the process standards should permeate mathematics instruction on a daily basis, and that they will be evident if effective instruction is going on in the classroom. Allow time for participants to make their lists on the index cards.	
	(Slide 46-49) **Credits and closing slides**	