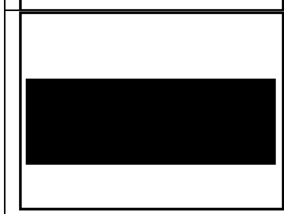
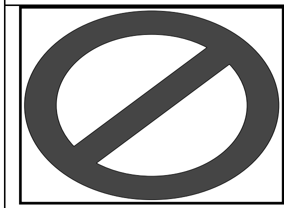
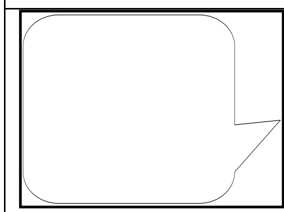
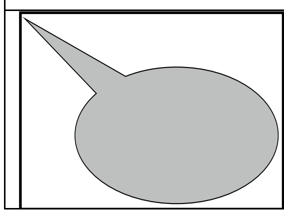
Materials

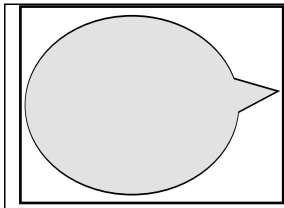
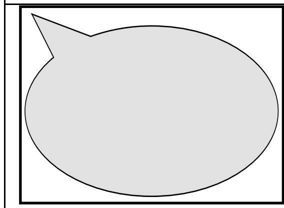
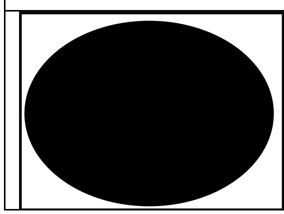
Computer, power strip	projector (LCD, overhead)	mini 10 frames (set per group)
power cord	overview video	Vis-à-vis, markers
SCOS	11x17 paper	workmats (5 & 10 frames, place value)
two-color counters (15/person)	markers	handouts
Arithmetic Rack Model	snap cubes or place value blocks (app 60/pair)	

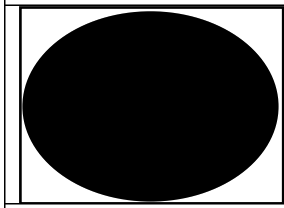
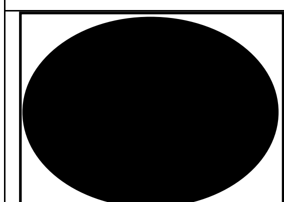
Note to leaders: There are a very large number of slides in this module. Many do not require more than a minute or so to read. However, planning your timing will be critical as you prepare to lead the professional development.

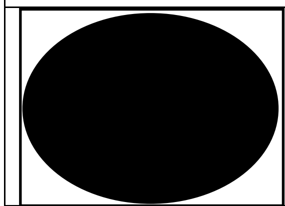
	(Slide 1) **Number & Operations** Welcome participants -Logistics Video Clip - Show video clip that introduces the professional development if this is the first module being used.	
	(Slides 2 – 3) **Football Fun** Read the story "Football Fun" aloud from the power point slides. Note that only slide 2 is shown on the left. Slide 3 has the end of the story.	
	(Slide 4) **Football Fun** Participants answer the questions, determining why the story doesn't make sense. Ask participants to define what they think number sense means. Allow a brief discussion on what they think the components of number sense are. (Participants will probably mention experiences they had that helped develop number sense like playing games. Don't let them get off on how families today are different. Keep them focused – you may have to ask how they can tell a child has number sense…describe that child and his/her thinking.) The components of number sense will be highlighted after the big ideas.	

	(Slide 5) **PSSM** Go over PSSM (*Principles and Standards for School Mathematics*, NCTM 2000) recommendations for number sense. Note: These are the <u>national</u> recommendations from NCTM (National Council of Teachers of Mathematics).	
	(Slides 6) **SCOS and Big Ideas** Refer to the big ideas handout and the SCOS for grades K-2. Tell participants that they will discuss each big idea during this module. Have participants quickly read through the big ideas to start to become familiar with them. As you work through the module, participants will write in the objective numbers for their grade level for the number strand on the this handout.	
	(Slide 7) ***Standard Course of Study*** Note the progression of ideas throughout the grade levels. Do not let this time turn into a discussion of the objectives – what each one means, where things were moved, what was deleted, changed, etc. Be sure to state that the objectives will be revisited throughout the module so everyone will have a better understanding of the rationale behind the number objectives.	
	(Slide 8) **Big Ideas in Number** Introduce the first big idea in number. Note that this big idea is about number relationships and concepts students need to understand in order to become competent with numbers and operations.	
	(Slide 9) **Number Sense is Based On** Number sense is based on number, operations, and connections. These three categories are all intertwined throughout the number objectives and all of the other strands. Make the following points: • Number is embedded in every strand of mathematics. • When solving problems, students must apply their computation skills and then rely on their number sense to decide if their answers are reasonable.	

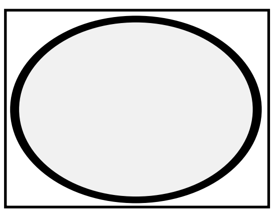
	• Problem solving is the tool that ties all of these components together.	
	(Slide 10) **Number Sense is NOT…** Stress the points that number sense can not be taught in a unit, a few activities or lessons, and it is not 1-2 objectives that can be taught and then checked off for students. Number sense is developed ALL year, EVERY year in every strand of mathematics!	
	(Slide 11) **Number Sense** Use these next slides to talk about components of number sense. This slide contains ideas of what number sense means – students with number sense have a good feel for numbers, think about numbers with flexibility, and understand relationships, magnitude, and the effect of operations on numbers. They also develop good operation sense. Ask a few participants to give a specific example for each bullet.	
	(Slide 12) **Number Sense** This is another way to describe students with number sense. Remind participants that fluency is the goal. Fluency is NOT speed. It is being accurate, being efficient in solving computations and resolving problems, and being flexible in approaching mathematical situations.	
	(Slide 13) **Number Sense** This slide gives examples for what students should understand if they know meanings of numbers. Ask participants to give other examples that are meaningful for their students, but don't dwell on these slides too long.	

	(Slide 14) **Number Sense** This slide gives examples of what students understand if they know number relationships. Point out that these relationships also are true for rational numbers, including positive and negative numbers, fractions, and non-terminating decimal numbers. Ask if participants have examples of relationships to share/add.	
	(Slide 15) **Number Sense** This slide gives examples for what students understand if they know magnitude (size of numbers). *In order to be successful with the mathematics tasks at your grade level what comparisons or understandings depend upon students' grasp of the magnitude of numbers?* Tell participants to give several examples to others at their tables.	
	(Slide 16) **Relationships Among Numbers** Introduce these three relationships among numbers: spatial relationships, anchors of 5 and 10, and part-part-whole. (These are not the only relationships that need to develop; these are ones that are particularly helpful.) Be sure to mention that many students in upper grades have gaps in their understanding because they have not developed these relationships. Activities that follow are examples of ways to develop these understandings during elementary grades.	
	(Slide 17) **Thinking Quantitatively** Students need to have experiences that encourage them to recognize quantities (groups) as a way of counting instead of always counting individual items. Subitizing is being able to identify small quantities without counting. Many subitizing activities help build spatial relationships with young children.	
	(Slide 18) **Spatial Relationships** Dot Cards are a vehicle for helping students recognize configurations of numbers. Talk about the points made on the slide. *What other common configurations of dots are elementary students familiar with?* (Both dominoes and dice use consistent dot patterns.)	

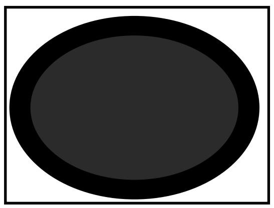
(note to leaders)	***Plan ahead for the next slide before you click.*** Tell participants they are going to try this out on the next few slides. You will click to the slide and leave the card on for 2 seconds then click to the next slide. Participants are to call out how many dots they see.	
	(Slide 19) **Spatial Relationships** Show dot slide for only 2 seconds!!	
	(Slide 20) **Spatial Relationships** Have participants tell the number of dots they saw. Then click back to the dot card (slide 19) and have a few participants describe how they knew the number of dots. Example strategies may include: I saw three dots, two dots, and 1 dot and knew that was 6 or I saw two groups of 3 dots and knew 3 + 3 is 6 or 3x2 is 6. Be sure to have all possible ways shared. Tell participants you are going to show another dot card and leave for 2 seconds again then click to the next slide.	
	(Slide 21) **Spatial Relationships** Show Dot slide for only 2 seconds!	
	(Slide 22) **Spatial Relationships** Have participants tell the number of dots they saw. Click back to the dot card on slide 21 and have a few participants describe how they knew the number of dots. Example strategies may include: I saw 3 and 2 or 4 and 1. Be sure everyone sees	

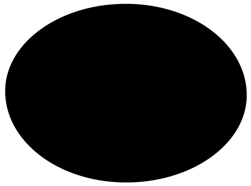
	how the dots can be counted in familiar dot configurations and not having to count each dot individually.	
	(Slide 23) **Spatial Relationships** Show Dot slide for only 2 seconds!	
	(Slide 24) **Spatial Relationships** Have participants tell the number of dots they saw. Click back to the dot card (slide 23) and have a few participants describe how they knew the number of dots. Example strategies may include: I saw 3, 3 and 2…I saw 6 and 2 or even I saw 9 and took away 1 to get a total of 8. Have the group focus on the questions you asked and the importance of students sharing strategies.	
	(Slide 25) **Spatial Relationships** Read points on the slide and have participants discuss how this activity will help students know number combinations and facts. Ask which grade level dot cards would be appropriate…hopefully they will see all elementary students need subitizing experiences if they have yet to internalize their facts and count in groups. Have participants add objective numbers to their handout related to the first big idea, noting how it is connected to their grade level objectives. They will add other objectives for the second big idea as they learn more about it during this section of the module.	

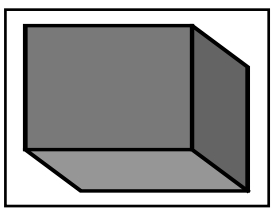
	(Slide 26) **Big Ideas in Number** Read through the second big idea. The term "composite" refers to a collection that can be thought of as one unit. So a nickel represents one unit that is a collection of five pennies. "Multiplicative" reasoning refers to the ability to think about and operate with collections as a unit. For example, children can describe the value of 20¢ as four nickels. This is multiplying 4 x 5 or adding 5 + 5 + 5 + 5. That is, the student is using the nickel as a composite unit rather than using 20 individual pennies.	
	(Slide 27) **Thinking Quantitatively** Have participants read the quote from Kathy Richardson on the slide and discuss their thoughts about it briefly with a neighbor. *What evidence do you have that supports this statement?*	
	(Slide 28) **Anchoring Numbers to 5 & 10** Explain the significance of anchoring to ten in our base ten number system. Young children must develop an understanding of the tens and ones structure of our number system and the ability to think about and work with numbers. Model asking questions about the frames on the slide. For example, "Looking at the five frame, how many more do I need to make 5?" Look at the first ten frame. How many more do I need to make 10? How do you know?" "Look at the second ten frame. How many dots are there? How many more do I need to make 10? How did you decide?"	
	(Slide 29) **Using Ten Frames** Pass out blank ten frame handouts and two-color counters. Tell participants to show the number 7. Ask: *How many more do you need to make 10?* Now tell them to show the number 4. Ask: *Did you clear the board and start over or just remove 3 counters? How do you think young children will show 4 after showing 7? What can you learn about children's thinking in tasks like these?*	

	Allow each table to practice using the ten frame mats and two-color counters by generating problems appropriate for K-2 students, focusing on the kinds of questions they would pose for students and what they might learn about students' thinking. Ask for a few participants to share problems/questions they created or ways they use the ten frame in their classrooms. Do not allow more than 5 minutes for this exploration. Make sure to make the connection that ten frame flashes can be used like the dot card flashes in the previous activity.	
	(Slide 30) **Arithmetic Rack** Show the homemade arithmetic rack. (There are two rows with ten beads on each row.) These can be used in the same manner as the ten frame with the same questions: Show the number 7. Now ask a participant to show the number 4. Ask: Do you need to clear the rack and start over or just move 3 beads over? Ask how far are you from 10? Invite a different participant to build the number 14. How might a student use the arithmetic rack to figure out what 8 plus 5 equals. Discuss how the rack is a different model that represents ten. Racks could be made with additional rows to show numbers greater than 20.	
	(Slide 31) **Part-Part-Whole Relationships** Understanding part-part-whole relationships is a major milestone in the development of number. Students need to know all of the different combinations for numbers to 10 and to be able to think about the different parts that make up any number. The next few slides suggest ways this relationship can be built in the classroom. Have participants count out 8 counters. Ask how a child would count out the counters. Point out to participants that nothing in the process of counting a set will cause children to focus on the fact that the total set could be made of two or more parts.	

	(Slide 32) **Part-Part-Whole Relationships** Have participants divide their 8 counters into parts. Have participants share the different ways they divided them. (4 and 4, 3 and 5, 2 and 6, 7 and 1 and the inverse)	
	(Slide 33) **Part-Part-Whole Relationships** Point out that it is important for students to build both parts and determine the total and to also be given the total and have to split it into two or more parts. Children will develop relationships with small quantities and then extend to larger quantities. Knowing part-whole relationships helps develop fluency with basic facts.	
	(Slide 34) **Part-Part-Whole Relationships** Students need to process these part-whole relationships visually, auditorally, and in written form. Stress that it is important to encourage students to verbalize what they are modeling.	
	(Slide 35) **Part-Part-Whole Activities** This slide illustrates familiar activities for young learners. These are described in numerous resources; two familiar ones are <u>*Developing Number Concepts*</u> by Kathy Richardson and <u>*Teaching Student-Centered Mathematics*</u> by John Van de Walle. Ask participants who have used the tasks to explain. (Directions are given below in case an activity is unfamiliar to the group.) Encourage participants who have not used the activities to try them in their own classrooms. <u>Description of Spill the Counters</u> -- Choose a designated number for the group. Have the children place that many counters in a canister. Shake and spill the counters. Using a recording sheet, have the students color in the appropriate number of counters. What do you notice about the number? Conversation may include, "I got 3 red and 2 yellow" or "5 can be 2 and 3."	

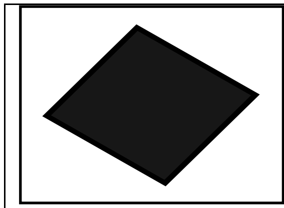
	Description of Cover Up – Choose a designated number and place that many counters under a cup or piece of tagboard. Next, pull some out from under the cup. Then the child will tell how many are hidden under the cup. For example, if there are 6 counters under the cup and 2 were pulled out, the child would respond, "2 and 4 is 6." Missing Part Cards – Using card stock, create a set of flip cards for numbers 1-10. On each flip card, write the designated number in the first space, and use the remaining two spaces for dots. Cover up the last set of dots (or the first set of dots). Have children respond and then flip the cover to check. This can be read as 5 is 3 and ? or 5 minus 3 is ?.	
	(Slide 36) **Part-Part-Whole Activities** Pass out two-column mats. Model having students use these mats and the two-color counters to complete the following activity. Place 7 counters on one side. *How many do I need to place on the other side to make 10? If 7 + 3 is the same as 3 + 7, how could I show this?* Model telling students that the whole is 12 and one part is 5. *What is the other part?* Remind teachers that students can use their mats to find all of the parts that make 9 (or any numbers). Note: Giving an individual student who is distracted by color some counters that are all one color (such as all green cubes) may help him/her focus on the number of counters on each group. Teachers who are familiar with *Math Their Way* and *Workjobs* (Mary Baratta-Lorton) or the Neufield math books will recognize these types of activities. They may wish to share similar tasks.	

	(Slide 37) **Extending Numbers** Part-part-whole activities should also be used for larger numbers. Use slide to show an example of 16. Ask participants why these two parts are decomposed by 10 and 6. Is that significant? Encourage a discussion on decomposing larger numbers into tens and ones…based on place value and why that is important.	
	(Slide 38) **Ten Frames and Two-Column Mats** Pass each group a baggie of mini ten frame cards. Have the tables work as a group to complete the tasks on the slide. **1.** Tell participants to break 83 into two parts, anyway they like. Discuss ways used in the room. You should see many ways where the number 83 has been decomposed by place value. Solutions may include (but are not limited to): 80 and 3, 70 and 13, 60 and 23, 50 and 33, and 40 and 43. **2.** Build 83 on one side of the mat. *What can you tell me about the number 83? How many do you need to put on the other for a total of 100?* Ask how these ten frame cards reinforce the idea of showing the parts that make up large numbers. *What other ways can these cards be used?* Ask participants how an arithmetic rack can be used to be another model for this idea. **Note:** Have participants bag their mini-ten frame cards but leave them close to be used again later in the module. Many participants may have experience with two-part work mats in which students focus on place value – ones on the right and tens on the left. In this activity, we are using the mats to represent parts of a whole.	
	(Slide 39) **Big Ideas in Number** Before moving on, ask participants if they want to add any objectives on their big ideas handout. This big idea will also relate to the next section of the module.	

	(Slide 40) **Big Ideas in Number** Introduce the next big idea in number. Understanding place value is fundamental to use of our number system. Give participants two or three minutes to read each part of the slide and explain to a partner what each bullet means. Before moving on, ask for volunteers to tell in their own words what the bullets mean as a way of summarizing the short table talk.	
	(Slide 41) **Place Value Assessment Task** Ask participants to think about how they would respond to the prompt. Have participants tell their partner how they think children would respond to it. *What might their drawing look like?* The assessment task, developed in the work of Constance Kamii, asked students to count out a number of counters (for example, 16), indicate which counters were meant by the 6 (interviewer points to the digit) and which counters was meant by the 1 (interviewer points to the digit).	
	(Slide 42) **Place Value Assessment Task** *What do you know if a student circles the eight counters correctly when asked to indicate what 8 means?* A student might circle 10 counters or he might circle 1. *If the child circles 1 counter, what place value understanding appears incomplete?*	
	(Slide 43) **Place Value Assessment Task** As an assessment, a teacher might ask the student to do the task on the screen. Prepare 5 bars of ten cubes and 3 more cubes. Show the cubes and ask participants to think about how students would respond to this activity and how they would determine how many cubes they had altogether. *Would this be different according to age or according to experience? Would the student's response likely be the same or different if the teacher places 4 tens and 13 ones on the able?*	
	(Slide 44) **Place Value** Possible ways students may count:	

	1 – Counting by ones: A student might count each individual cube by ones. This is the method that students begin with, and initially, the only way they can "tell how many." This is a "counting all" strategy. 2 – Counting by groups and singles: A student might count each group of ten as 1, 2, 3, 4, 5 bunches of 10 and 1, 2, 3 singles. Saying there are 5 bundles or tens and 3 ones. The student has now moved to counting a group of things as a single item. However, this counting method does not tell directly how many items there are. 3 – Counting by tens: A student many start counting all of them as tens and not distinguish if they are tens and ones. 4 – Counting by tens and ones: A student might count by tens -10, 20, 30, 40, 50 and ones - fifty-one, fifty-two, fifty-three.	
	(Slide 45) **Place Value** Note: The next five slides point out important components of place value. Point out the key role that counting plays in constructing base-ten ideas. Therefore, it is important that students have frequent opportunities counting sets of objects and counting them in a variety of ways. Children need frequent opportunities and a variety of models to count sets of objects in several ways in order to move to more sophisticated counting strategies and be able to understand groups of tens, hundreds, etc.	
	(Slide 46) **Place Value** Children must develop the idea of groups of tens for themselves. This idea can not be constructed for students- they must do it themselves. They need multiple opportunities building and grouping sets of ten and recognizing that this is a unit of ten and can be counted as one composite unit of ten. This takes time and multiple counting opportunities to develop!	

	(Slide 47) **Components of Place Value Understanding** Students need to move from one place value model/notation to another with ease. ■ Groupings by tens ■ Standard oral names for numbers ■ Written forms of numbers	
	(Slide 48) **Components of Place Value Understanding** Recognizing different representations for equivalent groupings is a critical understanding for place value.	
	(Slide 49) **Components of Place Value Understanding** The oral names for numbers encourages thinking in terms of groups instead of single units. This is not true, however, for the teen numbers, and often is a source of difficulty for young children as they write 41 rather than 14 because of the way the number is spoken. Students may need to be made aware of these connections explicitly. The teaching of place value has to be intentional to connect representations of number, the symbolic form of number, and the oral language of number.	
	(Slide 50) **Components of Place Value Understanding** Another critical understanding of place value for students is the idea that we represent all numbers using only ten digits. The value of the number depends on its position. Students that understand patterns of the periods will be able to better understand place value in its written form. The pattern of ones, tens hundreds is repeated in each period: one thousands, ten thousands, hundred thousands; one millions, ten millions, hundred millions, etc.	
	(Slide 51) **Place Value Activities** Give each pair of participants one place value mat (there is a master in the handouts) and a set of place value blocks or snap cubes. Have them model numbers on the place value mats. Point out that you are walking them through	

	sample tasks that teachers will do in a much more expanded time frame in their own classrooms. Encourage them to share observations of things that are likely to arise in classroom settings. For example: Build the numbers 46, then 13, then 24, then 37. *With each new number would students clear the board or add/subtract blocks accordingly? What other things about students' thinking could you observe during this activity? What questions might you ask as students model the numbers?* Next instruct participants to build the number 18 on their place value mat. Have them add a second number such as 35, splitting the mat horizontally. Ask a volunteer to demonstrate how the mat could be used to help arrive at the sum. *What are the differences/benefits of using this type of place value mat instead of the traditional two-column mats?*	
	(Slide 52) **Models for Place Value** Some place value models are more concrete than others, though the value of any manipulative lies in the meanings students construct. (The slide shows models from most concrete on the left to most abstract on the right.) For example, if a child does not see the rod as one unit of ten (seeing it instead as just a long block and counting it as one along with the unit cubes), the blocks are not helpful in modeling place value aspects of numbers. Labinowicz identified what he felt was the most concrete to the most abstract representation for number: loose objects that could be spilled out of a cup to turn from a ten back into 10 ones immediately, objects that can be bundled or snapped together and easily ungrouped, beansticks that students create (not in his original list) but must be traded, scored base ten materials, un-scored base ten materials, chip trading, and finally numbers.	
	(Slide 53) **Place Value Tools** *Why might it be important to use a variety of models for developing place value understandings? What other tools/models can be used to help build place value understandings?* Possible additional tools that may be mentioned are hundreds boards, money, etc.	

	(Slide 54) **What Number Is It?** Tell participants that you are going to model the game "Guess My Rule" with the 1-100 chart. This game also appears in the Geometry and Algebra modules. It is a simple game that is easy to play and can be used to teach several concepts. Instruct participants to study carefully the spaces that are covered to see if they can figure out the rule. Instead of having them tell the rule, have them name other numbers that could fit the rule. (The rule is multiples of 7.) Once several participants have identified additional numbers, discuss the rule. Have participants brainstorm other rules to try. Some possibilities might include – multiples of 10 or 5, numbers with 6 in them, even or odd numbers, digit in the tens place is one greater than the digit in the ones place. If time, participants can play the game with a partner.	
	(Slide 55) **Guess My Rule** Show the slide for a slightly different approach to the game. Tell participants to generate ideas of what the rule could be. Notice how open-ended this is, allowing all participants (in the classroom, students) to engage in the game. They can revise their guesses after seeing more numbers that fit the rule. *What might the rule be?*	
	(Slide 56) **Guess My Rule** Have participants revise their guesses after seeing another number that fits the rule. *Which rules have you eliminated and why? Which rules are still possibilities?*	
	(Slide 57) **Guess My Rule** Again, have participants revise their guesses after seeing another number that fits the rule. *Which rules have you eliminated now and why? Which rules are still possibilities? Are there multiple possibilities?*	

(Slide 58) **Guess My Rule**
Tell the participants that no other numbers on the hundreds chart fit the rule (but there would be some numbers on a 200s chart). *Does anyone know the rule?* (The rule is "the digit in the ones place is twice the digit in the tens place.*") Are there any other rules that could also fit only these 4 numbers? What other numbers, not on the chart, would fit the rule? What about the number 124?*

(Slide 59) **Big Ideas in Number**
Before you talk about this slide, give participants a few minutes to add objectives to their big idea handout. Suggest that they keep the handout before them since this slide and the next introduce big ideas in number. Read (or have a participant read) the information on the slide. Then display the next slide.

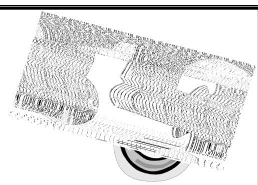
(Slide 60) **Big Ideas in Number**
again, read the information. Ask participants to talk at their tables and restate the information in their own words. Make certain that everyone understands the ideas. Both the fourth and fifth big ideas are illustrated in the next portion of the module.

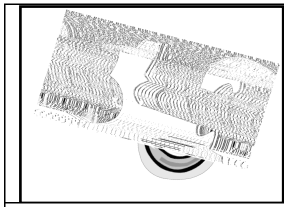
(Slide 61) **Teaching Addition & Subtraction**
Introduce the main teaching strategies for helping children master addition and subtraction (contextual problems and models). Experiencing a variety of models and problems that are contextual help children move through the levels of solution strategies and become proficient at solving expressions and problems.

(Slide 62) **Teaching Addition & Subtraction**
Students need opportunities to solve many different types of problems as they are learning to add and subtract. This slide indicates one form of guidance in selecting problems and tasks as teachers are planning lessons.

- Routine drills with no context – ex. 7+9
- Routine applications in a context – ex. Seven boys and 9 girls are riding on the bus. How many children are on the bus?
- Multi-step drills with no context – ex. 5 + 8 – (3 + 3) or 4 + ? = 6 + 2

	• Multi-step applications in a context – ex. Marty collected 5 baseball cards and 8 basketball cards. He gave 3 cards to his friend. How many cards does he have left? • Non-routine – The candle maker has 16 candles. He wants to put them into packages of 2 or 3 candles. What packages could he make?	
	(Slide 63) **Problem Based Lessons** Go over the points on the slide explaining the focus of problem-based mathematics lessons. The problem solving module will go into greater detail on classrooms that support problem-based learning.	
	(Slide 64) **Contextual Problems** This is an example of a contextual problem. It addresses number combinations and part-part-whole relationships. It is connected to children's lives and experiences. *What mathematics is addressed in this problem? At what grade level might you use this problem?*	
	(Slide 65) **Contextual Problems** Notice how this same contextual problem becomes appropriate for either students in higher grades or students who are ready for more of a challenge. *Is the same mathematics addressed in this problem? What different kind of thinking might be needed to solve this example? At what grade level might you use this problem?*	
	(Slide 66) **Problem Based Lessons** Read the quote to participants. Ask: *What is significant about this finding?* Some of this research came from the CGI –Cognitively Guided Instruction – studies showed that students' problem solving skills increased and the computation did NOT suffer at all when students were immersed in problem based situations instead of routine drill and practice.	

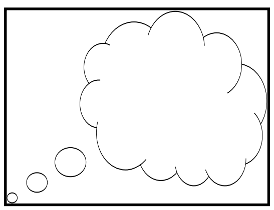
	(Slide 67) **How Would You Solve This?** Have participants read the Aidan problem and decide which operation to use to solve the problem. *What are the different ways that students at your grade level might solve the problem? Why do you need to have students explain the different approaches?*	
	(Slide 68) **How Would You Solve This?** Have participants read the Maggie problem and decide which operation to use to solve the problem. Have them discuss how students at their grade level would solve the problem.	
	(Slide 69) **Comparing Problems** Use this slide to have participants compare the two problems. *How are the problems are alike and how they are different? What different thinking can be discussed in the classroom when some students use addition and others use subtraction to get their answers? Are both approaches appropriate?* Have participants identify the key words in the problems, but don't dwell on them here. The next slides will focus the conversation on key words.	
	(Slide 70) **Key Words** Discuss the danger of using key words and why students are not likely to have "operation sense" if they are taught to look for key words instead of reasoning through the problem. Ask if children only look at the key words and don't read the problem, what are they likely to get as an answer? (7+15=22)	
	(Slide 71) **Key Words** *If children only look at the key words and don't read the problem, what are they likely to get as an answer to this problem?* (15 – 6 = 9) Test makers use problems like this - not to trick children but to check for true understanding. When students are not able to read the problems on their own (whether not yet having learned to read, not being fluent in English, or for other reasons), teachers should read the	

	problems to them. Lack of reading skills is never a valid reason to deny students opportunities to solve problems that require thinking and reasoning about the situations in the problems.	
	(Slide 72) **Basic Structures of Problems** There are 4 basic structures for addition and subtraction problems that can be sub-divided into 11 types. There is not time to explore these types in great detail. The purpose of sharing this information is to make participants aware of how slight changes in wording can impact the level of difficulty of a problem for students. Be sure participants understand the need to expose students to all 11 types of problems that appear on the next slides and to use problems that are contextual…not primarily "naked" computation problems.	
	(Slide 73) **Join Problems** Go over the three problem types for join problems on the slide. Have participants discuss how students might model each type of problem using manipulatives or a teaching tool mentioned previously. Notice that the missing addend changes place for each of these types. (8 + 4= _?_, 8 + _?__ = 12, _?__ + 4 = 8). The point we want participants walking away understanding is that students need to experience all three types in problem solving situations.	
	(Slide 74) **Separate Problems** Go over the three problem types for separate problems on the slide. *What are the three "separate" situations that these stories help model?* (12 - 4= _?_, 12 - _?__ = 8, __?_ - 4 = 8) The point we want participants walking away understanding is that students need to experience all three types in problem solving situations.	
	(Slide 75) **Part-Part-Whole Problems** Go over the two problem types for part-part-whole problems on the slide. *How might students model each type of problem using manipulatives or a teaching tool mentioned previously? What equation might students write for each of these? Are*	

	other equations also appropriate? Notice that the missing addend changes place for each of these types. For example, 4 + 8= _?__, 12 – 8 = _?__ (alternative for second example, 8 + _?__ = 12) These problems are almost like the join problems except there is not an action involved (no giving, getting, buying, etc…no action verb) Think about the difference in a situation presented in cartoons such as Dennis the Menace and Peanuts. In the former there is only one picture to tell the entire story. In the second example, there can be action from frame to frame.	
	(Slide 76) **Compare Problems** Read through the problem types for "compare problems" on the slide. *How might students might model/solve each problem? What pictures might students sketch to show the comparisons?* Compare problems present a good opportunity to think about quantitative reasoning - that is, the relationship of the quantities in the problem regardless of the numbers involved.	
(slide not shown)	(Slide 77) **Compare Problems** Go over the third compare problem type with this last problem example.	
	(Slide 78) **Solution Strategies** These are examples of ways that students might approach a problem. Direct modeling – a student needs to draw, use manipulatives, or count all of the objects one-to-one to determine a solution. This is a very low level strategy. Counting – a student counts on (hopefully from larger number) to determine the amount. This is a more sophisticated strategy than direct modeling. Recall/Number Facts – a student that has internalized facts or uses an efficient strategy to find the solution is using the most sophisticated strategy. An example could be a student that uses doubles plus one to solve 8 + 9. A student that knows 8 + 8 is 16 can use that to know 8 + 9 is 17 because it is one more than 16.	

	(Slide 79) **Other Solution Strategies** Additional strategies students may use to solve problems include: Reasoning quantitatively – a student is able to generalize the relationship of the numbers in the problem to the answer (separate from actually computing). Quantitative reasoning goes back to number sense. A student that understands relationships of numbers can apply that understanding to problem based situations. Quantitative reasoning is similar in structure regardless of whether the numbers are large or small. Applying an algorithm – a student follows an efficient set of steps to solve a problem. For example, to compute 124 – 87, students "borrow/regroup" in order to be able to subtract. Using problem solving strategies – a student may use one of many strategies such as; solve a simpler problem, guess and check, look for a pattern, make a table or chart, organize a list, work backwards, draw a picture, use equations, or any combination of the above.	
	(Slide 80) **Reasoning about Problems** These models show two different representations that can be used to think about any of the problem types that have previously been shared. Any part of the model can be the unknown and the numbers can be plugged into it in order to reason about what the missing quantity needs to be. Examples are on the next slide.	
	(Slide 81) **Reasoning about Problems** Notice how any part of the model can be the unknown. *Explain to your partner what each diagram represents.*	

	(Slide 82) **Models of Reasoning** Try this out! Have participants use any model to represent the following two problems. Work at their tables to discuss how the model may help students reason through the problems. • George has 12 coins. Eight of his coins are pennies, and the rest are nickels. How many nickels does George have? • George has 4 more pennies than Sandra. George has 12 pennies. How many pennies does Sandra have? Reasoning should always be part of every problem students solve. Always help students focus on what is known, what is not known, what is being asked, and if the answer makes sense for the story. For more information on problem types and helpful models for students, see the monograph on problem solving and quantitative reasoning by Randy Charles on the Partners CD.	
	(Slide 83) **Big Ideas in Number** Introduce the last big idea in number. Notice the emphasis on fluency as using mathematics with understanding. Have participants be ready to write in their handout how this big idea is connected to their grade level objectives as they learn more about it during this section of the module.	
	(Slide 84) **Developing Fluency** Point out that ideas developed throughout this module—number relationships, part-part whole experiences, subitizing, anchoring to 10, understanding magnitude, developing operation sense and focusing on problem solving and reasoning – all help move students toward fluency. Fluency is not fast or easy to develop. Number sense develops over time with LOTS of practice and many, many connections and experiences!	

	(Slide 85) **Developing Fluency** Efficiency is an important part of fluency. Students need to be able to be fluent with their math facts. They should be able to retrieve the facts mentally and quickly. "Memorization" is only one way to become fluent with number facts. Short mini-lessons can help develop strategies like some of the ones on the following slides.	
	(Slide 86) **Strategies for Addition Facts** Many of the basic facts can be learned easily with strategy practice. "One more than" facts can be connected to the next counting number. Patterns help students develop facts with zeros. Many children memorize doubles in the same manner they learn words to songs or rhymes.	
	(Slide 87) **Strategies for Addition Facts** Once doubles are mastered, then near doubles (doubles plus one) can be thought about easily by children who have had experiences decomposing (renaming) numbers. (Six can be thought of as 5 + 1) The "making ten" strategy relates back to working with ten frames. Example: 7 + 8. To make a ten 8 needs two more, so take 2 from 7 to make ten and then add the 5 left over from the seven to get an answer of 15. If students can decompose numbers to make a friendly number for a ten fact, then this strategy is easy. Ask participants how they would use this strategy to practice 6 + 8, 5 + 9, 3 + 8, 8 + 9.	
	(Slide 88) **Developing Fluency** Students need to be given opportunities to practice these strategies to internalize the facts. After addition facts are learned, students need to understand how subtraction facts are connected to addition facts and to be able to use these basic facts to solve two- and three-digit problems.	

	(Slide 89) **Addition Expectations** Students need to be able to add multi-digit numbers presented in both vertical and horizontal formats. *What kind of thinking might each format (horizontal and vertical) encourage?*	
	(Slide 90) **Multiple Solutions** Give several sheets of 11 x 17 paper to each group. Working by tables, have them illustrate 4-6 ways to solve the problem on the slide. At the bottom of the sheet ask participants to write a summary of the kind of thinking used in the particular solution method. Move around the room and be prepared to call on different participants to show their work and explain the strategy they used. When participants share their solutions to the problem, be sure to push them to identify what mathematics they were doing as they carried out different steps. For example, did they decompose some numbers because of "friendly" numbers? Did they use multiples of ten? Did they use their knowledge of money (ex. 25¢) to help them? Did they add an amount to one addend and reduce the other addend by that amount?	
	(Slide 91) **Compare and Contrast Solution Strategies** Teachers must make sense of their students' work. *How did each of these four students think about the problems? What understanding does each method illustrate?*	
	(Slide 92) **Compare and Contrast Solution Strategies** Identify the misconceptions or incomplete understandings illustrated in these three examples. *What questions would you like to ask each student?* Since "saying again slower and louder" is not likely to help remediate the situation, what types of experiences or conversations might be appropriate interventions or lessons for each student?	

	Tell participants that there is a discussion of some addition and subtraction strategies in their handout. These are for them to use later in discussions at their grade-level meetings. More work related to multi-digit addition and subtraction is planned for professional development through the Partners project for the 2009-2010 school year.	
	(Slide 93) **Relating Big Ideas and SCOS Objectives** Have participants work with a partner to place objectives with an appropriate big idea. Point out that some big ideas may relate to multiple objectives.	
	(Slide 94) **Reflecting on Our Work** Give participants index cards and ask them to complete one of the prompts on the slide. Ask them to write several sentences in their responses – not just a single word or phrase. Before you collect the cards, have participants write on the back the site number for your Leadership Team, your school district, and the date. Collect the cards and turn them in to your LEA contact to be mailed to the project's evaluator, Anita Bowman.	
	(Slide 95-97) **Credits** These are not pictures.	