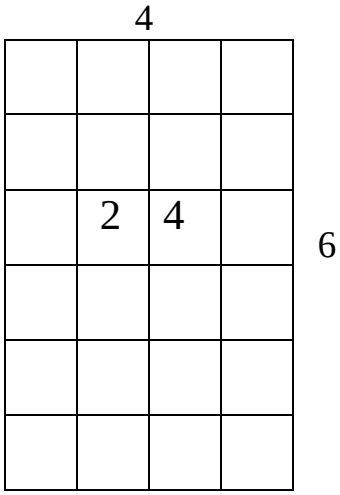
Arrays – a Visual Model

In elementary grades, students extend their understanding of multiplication through their work with arrays. Students are introduced to arrays as rectangular arrangement of objects in rows and columns - a visual image that supports students' understanding of multiplication. Students use rectangular arrays to represent the relationship between a product and its factors.

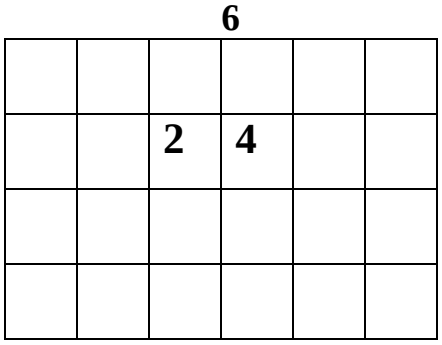
Activity:
Students are presented a problem set in a context. They are given square tiles or cubes, scissors, and one inch grid paper. Students work in groups of 3 or four to find solutions.

Problem:
The students in Mr. Jernigan's class want to figure out all the different ways they can arrange the 24 chairs in their classroom. The chairs must be arranged in straight rows and each row must have the same number of chairs with no chairs leftover. How many different arrangements can you find? How many chairs are in each row?

Brad's arrangement: 6 rows of chairs with 4 chairs in each row.
The teacher draws a 6 x 4 rectangle showing squares that represent each chair.



Students search for a different arrangement with the same dimensions as Brad's arrangement.



Students compare and contrast arrays with the same dimensions. Rotating one array allows students to see the arrays are congruent.

6 x 4 = 24 (Six row with four chairs in each row equals 24 chairs)
4 x 6 = 24 *(Four rows with six chairs in each row equals 24 chairs.)*

Students continue to share possible arrangements of 24 chairs. They discover that each arrangement of 24 can be rotated to create a different arrangement with the same number of chairs. Many students struggle with arrays as they have difficulty understanding how a square can count as a row and a column simultaneously. They are very surprised that it doesn't matter if they count the number in each row or the number in each column, the total number of chairs is always the same.

Teacher uses students' prior knowledge of the commutative property of addition to make the commutative property of multiplication transparent to students.

Students learn that the dimensions of the rectangle are factors of 24. With student participation, the teacher records factors of all arrangements for 24 chairs.

1 x 24	2 x 12	3 x 8	4 x 6
24 x 1	12 x 2	8 x 3	6 x 4

Students find eight different arrangements/rectangles for the 24 chairs. They figure out there are 4 different rectangles if they do not count the reversals. Students use their arrays to find all the factors of 24 (1 2 3 4 6 8 12 24) Students identify factor pairs for 24 and use reasoning to answer, "How do you know you have all the factor pairs for 24?" As they record their answers, students connect factor pairs to multiplication combinations for 24. An extension might be to identify the factors of 24 that are also factors of 48.

Students discover they can skip count by each of the factors and land exactly on 24. They test other numbers and discover there are no other counting numbers they can skip count by and land on 24. Students refer back to skip counting charts, completed earlier. Students identify numbers which have 24 as a multiple and compare their findings to the factors of 24. A factor of 24 is any whole number that divides 24 into equal groups with a remainder of zero. Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. Twenty four is a composite number as it has more than two factors.

Activity:

Materials: color tiles, 1 inch grid paper, sharp markers, scissors

Ahead of time, teacher places a long piece of bulletin board paper in a space accessible to students. Teacher list numbers 1=50. Numbers are spaced to allow students to tape rectangles for each number.

For this activity, students decided two arrays that are congruent will be considered as the same array.

- Working with partners, students are asked to find all of the arrays for their assigned numbers from the set of whole numbers 1 - 50).
- Students draw arrays on grid paper. Label the dimensions and the product (area) of each rectangle.
- Students cut out all arrays for each assigned number.
- Students tape each of their rectangles under the correct number. Example: Rectangles for 12 are 3 x 4; 2 x 6; 1 x 12

One group asked, "Does a 4 x 4 count? It makes a square, not a rectangle?" This question provides an opportunity for students to identify properties of a rectangle and properties of a square to understand why mathematicians include a square as a rectangle.

After all rectangles are taped under numbers 1-50, Students sit with partners or in Teams of 3 or 4 to discuss and record all pattern they find in the class representation of rectangles of number 1-50. Students identified square numbers as the result of multiplying a whole number times itself. Students extended their list of square numbers to larger numbers and were able to justify why square numbers have an odd number of rectangles. Students listed all the numbers with one rectangle and found examples to support the definitions of prime and composite numbers. Students used arrays to identify and describe number relationships and multiples.

Students use the array model to reason why doubling and halving a pair of factors will not change the product. As students solve multi-digit multiplication, they draw open or unmarked arrays. 38 x 25 **25**

38	20	5
30	600	150
8	160	40

Product = 950 There are many ways students can decompose 38 x 25.

Arrays support students' understanding of the relationship of multiplication and division.