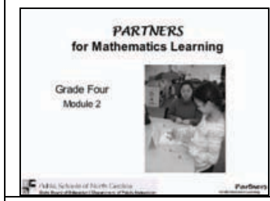
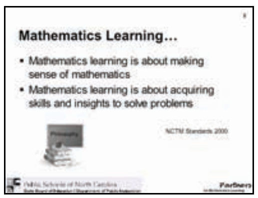
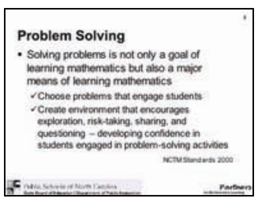
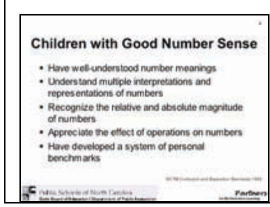
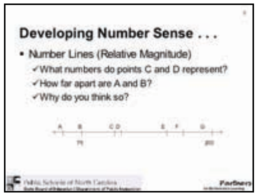
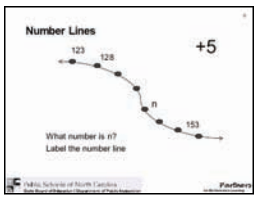
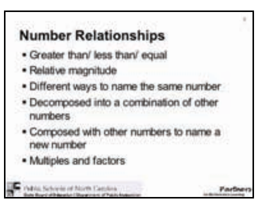
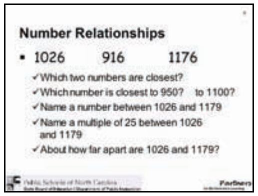
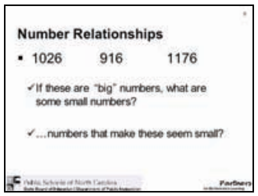
General Materials and Supplies:
Transparency of Four's a Winner gameboard, paper clips, and transparent markers in 2 colors (optional)
Calculators, some programmed for order of operations, e.g., TI Math Explorer, and some not, e.g., TI-108
3 number cubes per table (2 with numbers 1-6, 1 with numbers 4-9)
Index Cards 10-sided die with numbers 0-9 (one per table)

Slide	Tasks/Activity	Personal Notes
	(Slide 1) **Partners Grade 4 Module 2** Number and Operations This module continues the number and operations strand, focusing on multiplication and division. Ask participants to look for the process standards, especially problem solving, and algebraic thinking in the number and operation activities.	
	(slide 2) **Mathematics Learning…** The two points, which are dual goals of the NCTM 2000 Standards, are repeated from the Introduction to Module 1. Remind participants that these are also goals for NC's curriculum and are focal points of the activities and discussions within the workshop modules. Remind participants of the introductory handout that presents more on the philosophy of teaching and learning.	
	(slide 3) **Problem Solving** This is an important idea repeated here in different words than in Module 1 – that instruction in mathematics should be based on problem solving. It involves the selection of meaningful problems that are engaging for the children and provide them the opportunity to use prior knowledge and skills to develop new ideas and skills. It also involves creating an environment where children are safe to explore, make mistakes, learn from them, share their thinking, question ideas, and therefore develop new learning.	
	(Slide 4) **Children with good number sense** This is what children with good number sense "look like" – how they demonstrate their good number sense. The next few slides suggest ways to help children develop and extend good number sense.	

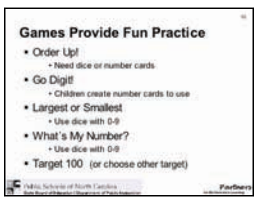
	(Slide 5) **Developing Number Sense…** Number lines are valuable tools to help children develop a sense of the relative magnitude of numbers. Using this number line, ask the questions on the slide. Mention this additional strategy: using a blank number line with starting and ending points marked (e.g., 0 at one end and 100 at the other), mark a point with a ? and ask the children to guess the number. For incorrect guesses, mark the spot where that number would be. The children should be able to narrow down their guesses until someone guesses the correct number.	
	(Slide 6) **Number Lines** Number lines needn't be straight, and needn't always start at 0 or 1, although they do need a consistent scale. On this number line the +5 pattern above 0 would be a 2, 7 pattern, not a 0, 5 pattern. Highly developed number sense will allow children to count by tens off the decade and to count by other numbers starting at a number that is not a multiple of that number, as in the number line above. Counting around the room by 3's for instance, but starting at 19 (or another non-multiple of 3) can help children see new patterns and focus on number relationships they may not have thought of before. Point out that looking for such patterns is an algebra connection involving numeric growing patterns.	
	(Slide 7) **Number Relationships** Go over this list of some of the kinds of relationships that can be found in instruction involving number. Ask participants for examples of each or give examples as needed. For example, 48 can be decomposed into 40 + 8, 4 tens and 8 ones, 24 + 24, 10 + 10 + 10 + 10 + 8, etc. These are also different ways to name the same number, but this can be extended to include 50 – 2, 4 dozen, 52 less than 100, and the like. Relative magnitude- 10 is it a large or small number in relation to 300? Or 0.003?	



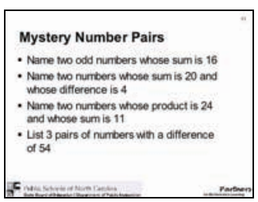
(Slide 8) **Number Relationships**
Ask the questions on the slide. As participants answer, ask how they know. Use these relationships to talk about inequality. The numbers could be written with > and < signs. The third bullet describes a number greater than 1026 and less than 1179. Ask how many whole numbers fit that description. How many of those numbers are even? …are multiples of 5? Ask participants for other extensions of these questions. Plotting these numbers on a number line could provide another model for understanding the relative magnitude of these numbers.



(Slide 9) **Number Relationships**
Ask participants for responses. Questions like these can help children develop a greater sense of number relationships.



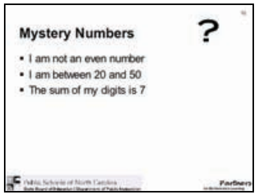
(Slide 10) **Games Provide Fun Practice**
This slide references these games in the handouts: *Order Up!, Go Digit!, Largest or Smallest?, What's My Number?*, and *Today's Target*. Assign a game to each group. Have them read directions in the handout and play a round or two of their assigned game. Be sure to point out that all the games or activities are in the handouts for them to use later. Note that *Toda y's Target 100* could be changed to be any number (today's date, 1000, etc.) Point out, also, the activity, *Compatible Numbers*, which utilizes "friendly" numbers to reach a sum of 50.



(Slide 11) **Mystery Number Pairs**
Note that each clue is referencing a new number pair (this is not a list of clues that must all be satisfied with one pair. Possible solutions: (9,7), (8,12), (3,8).

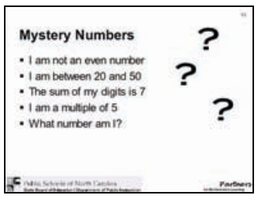
The notion of "the same difference" (the second and fourth bullet) is critical in students' understanding of subtraction. For example, for "502-367" is easily solved as "499-364".

The fourth direction has an infinite number of possible solutions. Examples (1,55), (45,99), (100,154), (150,204). What would you look for in a child's responses to tell you about their level of sophistication with number relationships?



(Slide 12) **Mystery Numbers**
Given these three clues, have the participants make a list of possibilities (25, 34, 43).

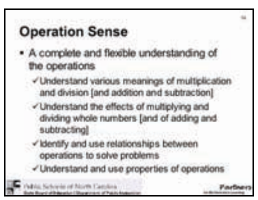
Suggest that with children, one clue should be shown at a time. Children can think of a number that fits, then revise the number if necessary with each additional clue. Then show the next slide.



(Slide 13) **Mystery Numbers**
From the three possibilities, participants should recognize the mystery number as 25.

Discuss the use of mystery number clues as a good way to use vocabulary related to number. Then have each table write a set of clues for a mystery number on large paper.

Post them around the room for other groups to try at break. See "Mystery Numbers and Other Number Puzzles" in handout.



(Slide 14) **Operation Sense**
Remind participants of these meanings of operation sense mentioned in Module 1. Expand on the idea of operation sense by discussing the following ideas related to this slide:
What are meanings of multiplication?

- repeated addition
- groups of (equal groups)
- jumps on a number line
- multiplicative comparisons (e.g., four times as many as…)

Models for multiplication include array model, groups of (equal sets), number line
Solution strategies for multiplication:

- repeated addition
- skip counting
- doubling
- using partial products
- using five-times and ten-times
- doubling and halving
- factoring and grouping flexibly

using properties of multiplication

What are meanings of division?

partitioning (total and number of groups known) – equal groups or fair shares

measurement (total and number in each group known) – repeated subtraction

What are the properties of operations?

commutative

distributive

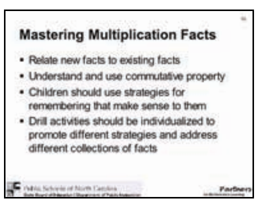
associative

identity properties (of 0 for additive identity and 1 for multiplicative identity)

(Slide 15) **Mastering Basic Facts**

A foundation of operation sense is work with basic facts. Mastery of basic facts must be built in part on understanding the operation involved. Note that children who have not developed mastery of basic facts by this level are children who have not developed efficient methods of finding a fact answer.

Mastery will not occur through drill of inefficient methods. The "rule" is "Do not subject any student to fact drills unless the student has developed an efficient strategy for the facts included in the drill." (Van de Walle p. 94)

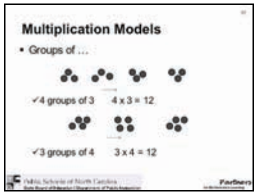


(Slide 16) **Mastering Multiplication Facts**

Emphasize that by 4^{th} grade, students have been exposed to the multiplication facts and hopefully have been working on strategies for remembering them (rather than just memorizing them). So the important focus for 4^{th} grade is to continue helping children use strategies for remembering facts and to use drill on an individual basis as children are ready to practice facts that have meaning for them.

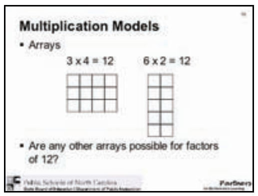
To demonstrate relating new facts to existing facts, have participants choral count by 3's beginning at 0. Record the numbers on the board as they say them, up to 36. Then ask them to look for patterns in the list. They may see the odd-even pattern, among others.

Then have them count by 6's up to 36. Record this list next to the 3's list. Have them compare the two lists. They should see that the even numbers in the 3's list are the numbers in the 6's list. The addition of a count and listing by 2's would be helpful for some children. Activities like this can help children see the relationships between tables.



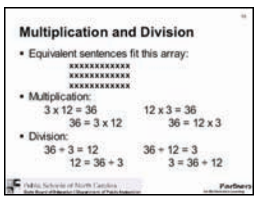
(Slide 17) **Multiplication Models**
One important model for multiplication which will have been used in third grade is the "groups of…" model in which children divide sets of objects into equal size groups. Two examples are given in the slide.

This slide also demonstrates the commutative property of multiplication.



(Slide 18) **Multiplication Models**
Another important model is the array model. This model representing simple multiplication situations is foundational to understanding array models representing multiplication situations involving larger numbers.

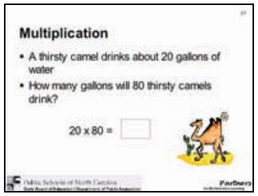
What other arrays are possible for factors of 12?



(Slide 19) **Multiplication and Division**
All of these equations can be written to describe the 3 by 12 array. Use this slide to discuss commutative property of multiplication, but not division; and emphasize the need for children to have the flexibility of writing these equations a variety of ways.
Also, emphasize again the function of the = sign as "is the same as" rather than "the answer is coming."

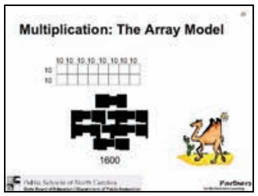
(Slide 20) **Practice Can Be Fun**
Four's A Winner is a strategy game involving both multiplication and division facts practice. See handout for gameboard and directions. If you have time, play the game using a transparency of the gameboard and transparent markers. You could play the group or divide the group into two teams who play each other.

Emphasize the point that practice using games can be not only fun, but more effective than worksheets.

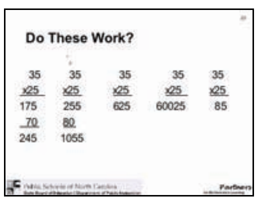


(Slide 21) **Multiplication**
Ask how to solve. Most participants may say "multiply 2 x 8 and then add two zeros," which gives the correct result of 1600. That is the shortcut, but why does it work?

Click to the next slide for a model that shows why 20 x 80 = 1600.



(Slide 22) **Multiplication: The Array Model**
This slide shows the array model for multiplying multiples of 10 (100, 1000, etc.). Rather than a 1x1 box, this model factors the number into groups of 10 so the rows show 80 as 8 groups of 10, and there are 20 of these rows with the 20 factored into 2 groups of 10. Symbolically this can be written as (2 × 10) × (8 × 10). The associative property allows for looking at this as (2 × 8) × (10 × 10), which defines the number of sections (2 x 8) and the number of gallons in each section (10 x 10). 2 x 8 = 16 sections. 10 x 10 = 100 gallons in each section. Thus, 16 x 100 = 1600. This concept/skill is essential in solving multiplication problems involving 2-digit numbers that are not multiples of 10.
Students will need background in understanding computations by powers of 10: 16 x 100 = 1600; 16 tens – 160; 3 hundreds = 300; 34 hundreds = 3400; 27 thousands = 27000, etc.
See handout "Animal Facts" for examples of this.



(Slide 23) **Do these work?**
None of these work. Have participants analyze what the children did and what the errors are. Point them to handout "Do these work?" where they can make notes about these solutions. Have participants explain and highlight the errors.
Errors include:

a) Second partial product: 2 x 35 rather than 20 x 35
b) 5 x 35: multiplied 5 x 5, got 25, "carried" the 2, then multiplied 5 x (3+the "carried" 2, or 5); then multiplied 2 x 5 for 10, and multiplied 2 x (3 + the "carried" 1, or 4)
c) Multiplied 5 x 5 = 25; 2 x 3 = 6, put the 6 in the hundreds place
d) Multiplied 5 x 5 = 25, 20 x 30 = 600, paid no attention to place value in writing the product.
e) 5 x 5 = 25, put 5 in ones place and "carried" the 2; then multiplied 2 x 3 = 6 and added the carried 2 for 8 in tens place.

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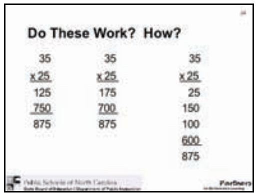
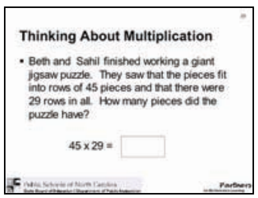
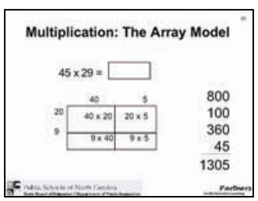
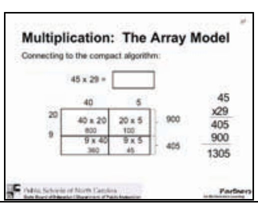
<tr><td>
</td><td>(Slide 24) Do these work? How?
Each of these does work and can also be found on handout "Do these work?" Ask participants to explain why each works.

The first example multiplies 25 by 5 first, then by 30, sort of an upside down version of the traditional algorithm.

The second multiplies 35 by 5 and then by 20, the traditional algorithm.

The third multiplies each part of the problem separately: 5 x 5, 5 x 30, 20 x 5, and 20 x 30, to show all the partial products.</td><td></td></tr>
<tr><td>
</td><td>(Slide 25) Thinking About Multiplication
After participants read the problem, click to next slide to show the array model for solving this kind of multiplication problem.</td><td></td></tr>
<tr><td>
</td><td>(Slide 26) Multiplication: The Array Model
This area (array) model of multi-digit multiplication shows how the partial products are determined. It utilizes the distributive property. This is a powerful piece for helping students understand multi-digit multiplication and to "see" how the partial products are derived.

Participants should remember this model from last summer's work. It also connects to the traditional algorithm – See the next slide.</td><td></td></tr>
<tr><td>
</td><td>(Slide 27) Multiplication: The Array Model
This slide shows how this array model connects to the traditional compact algorithm. The traditional algorithm is an efficient method of multiplication, but it is important that instruction is designed to lead up to it so that children understand why it works, not just how it works.

See handout "The Array Model" for examples of this model. You may choose to have</td><td></td></tr>
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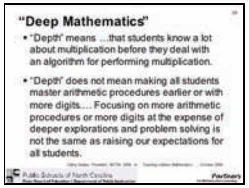
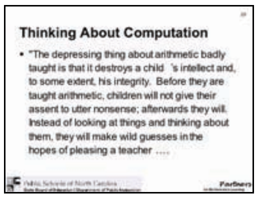
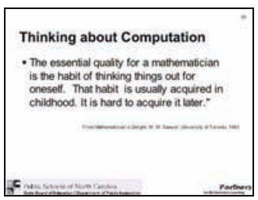
participants try one or more of the problems in the handout using this model.



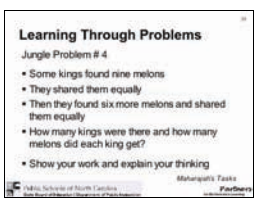
(Slide 28) **"Deep Mathematics"**
These quotes are from an article in an NCTM newsletter called "Hard Arithmetic is Not Deep Mathematics" by Cathy Seeley, president of NCTM in 2004. This article can be a powerful piece to share with parents to help them understand why math class may be different from what they expect or remember. Emphasize that we want children to do deeper mathematics and deeper thinking to develop deeper understandings. Deeper understandings is the higher level expectation – not bigger numbers or the early learning of rote procedures.



(Slide 29) **Thinking About Computation**
Go to the next slide for the rest of this quote.

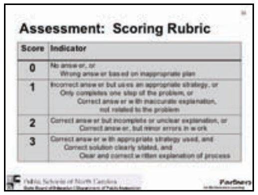


(Slide 30) **Thinking About Computation**
This quote is another way to say that teaching traditional algorithms too early destroys number sense. This was said 65 years ago, and is still true today – and perhaps even more important because of how important and valuable critical thinking and problem solving will be for our children in this 21st century.



(Slide 31) **Learning Through Problems**
Have participants do this problem. Allow them to work together, but each person has to produce a result to be evaluated.

After they have had time to work, talk about what a top quality solution would include – which means especially that the work and the explanation take into account that they divided the melons in two separate occasions, so adding the melons together and then dividing, while it might give the correct solution (3 kings who each get 5 melons), it does not represent the problem and can also give an incorrect solution of 5 kings who each get 3 melons. The correct solution shows that the 3 kings each got 3 melons from the first division and 2 from the second division, and explains why 3 kings works when 6 or 9 or some other number don't work, unless you allow for cutting the melons into equal size pieces. Some children will do just that and it is a reasonable solution as well, as long as it works mathematically.

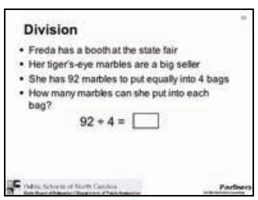


(Slide 32) **Assessment: Scoring Rubric**
This is one possible holistic rubric for scoring the problem. Note that an integral part of solving the problem is a communication piece – explaining the thinking that went into the process of solving the problem.

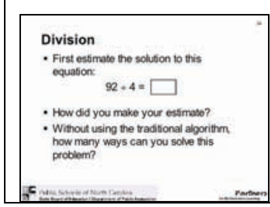
Look at student work in handout ("Melon Problem: Student Work," 3 pages). Use the rubric to score. Discuss scoring at tables; then as a whole group, share thoughts about student strategies (what surprised you, what would you have predicted, etc.), and scoring these solutions. Choose one or two to focus on and more as time permits.

Point participants to other "Jungle Problems" in handout and a rubric ("Rubric for Jungle Problems") that students can use to self-assess solutions. This can be done after a teacher has scored the solution, made comments about the solution for the student to learn from, but before the teacher shares his/her score with the student.

Student self-assessment is an important aspect of meaningful learning.

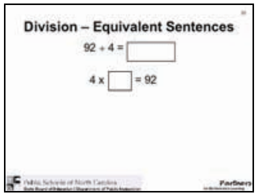


(Slide 33) **Division**
Have participants read the problem, then click to the next slide.



(Slide 34) **Division**
Have participants first make an estimate for a solution to this division task. Ask them how they arrived at the estimated solution. Then ask them to find ways to solve this problem <u>without the traditional algorithm</u>.

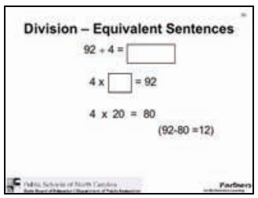
Use the "Number Talk" directions for a whole group discussion; or share at tables first, then with the whole group. See the next slide for an example of an alternative solution strategy.



(Slide 35) **Division – Equivalent Sentences**
Use this and the following two slides to talk through this method step by step. Giving the numbers a context helps. Children can give you the story for the problem.

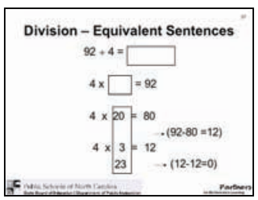
Suppose this one is "The PE teacher has 92 foam balls that need to be put into 4 bins with an equal number in each bin. How many balls will go in each bin?"

The first step is to write an equivalent multiplication sentence, in this case 4 times "something" equals 92.



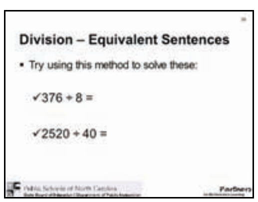
(Slide 36) **Division – Equivalent Sentences**
Begin to solve the multiplication sentence by finding multiples of 10 foam balls that could go into each bin. In this case there are enough balls for two groups of 10 or 20 balls to go into each bin. That takes care of 20 x 4 or 80 balls, which leaves 12 balls to go into the bins.

Using this method allows children to try smaller multiples, e.g., 10 balls in each bin, then 10 more balls in each bin. The exact number of tens doesn't have to be found on the first try.



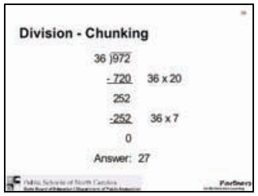
(Slide 37) **Division – Equivalent Sentences**
Complete the problem: Since 4 x 3 is 12, 3 more balls can go into each bin, making 20 + 3 or 23 balls for each bin. The process is so much more meaningful and easy to understand when put into a context and talked through in terms of that context.

The next slide provides two problems for participants to try using this method.



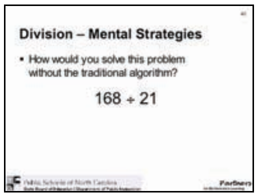
(Slide 38) **Division – Equivalent Sentences**
Have participants try the equivalent sentences method to solve these two division problems. (Solutions: 48 and 63)

"One Alternate Strategy for Division: Missing Factor" in handout shows an annotated example of this method.

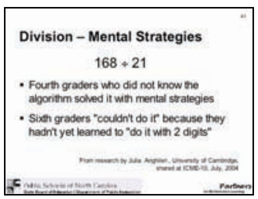


(Slide 39) **Division - Chunking**
This is a different notation, but the process is similar. In this case, in the first step, 20 groups of 36 are taken care of first, leaving 252 which is 7 groups of 36.

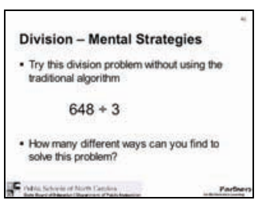
Ask participants for a story and talk about the problem in terms of the story.
Ask participants to talk at their tables about how their students might "chink" in a different way.



(Slide 40) **Division – Mental Strategies**
Elicit responses from the participants. Then show the next slide.

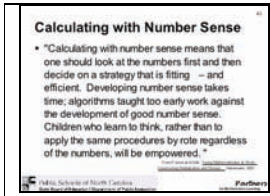


(Slide 41) **Division – Mental Strategies**
This result from research points out again that teaching children to lean too much on algorithms results in a lack of number sense and a lack of confidence. This is really not a difficult division problem if you look at the numbers carefully.



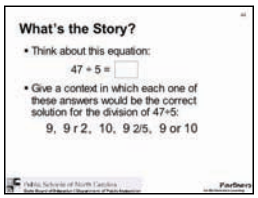
(Slide 42) **Division – Mental Strategies**
Have participants try this problem without using the traditional algorithm. Let them share their strategies.

(Slide 43) **Calculating with Number Sense**
This quote is from Young Mathematicians at Work: Constructing Multiplication and Division, Heinemann, 2001. It underscores the need to help children develop strategies derived from the numbers they are working with, not just looking at single digits or following rote procedures.



We must ask our children to think, reason, and solve problems, if we are to prepare them for the unknowns of this 21st century. They will have problems to solve that we cannot even imagine. We must not cheat them out of the extraordinary opportunity that mathematics presents to develop the skills they will need to solve those problems.

Memorizing facts and procedures without meaning will not "do the trick" to prepare them to meet these challenges. Thinking through problems, working collectively, evaluating and justifying solutions, learning to take risks and learn from mistakes – all these will help children be ready for whatever they may face in the future. We have to be willing to take the risks involved in helping children develop these thinking and risk-taking skills.



(Slide 44) **What's the Story?**
Remind participants of this kind of activity from last summer. Don't spend a lot of time on this review, but do note the importance of giving children opportunities to interpret the remainder in a division solution.

Points might include the following: Some kinds of problems require that the remainder be ignored. "Johnny had 47 pictures to put in an album. If he put 5 on each page, how many full pages would he have?"

Others require a solution that adds one to the whole number quotient. "The fifth graders were going on a field trip. If parents were driving and each car could hold 5 children, how many cars would they need?"

In other cases, the quotient can be expressed as a mixed number. Jordan had 47 feet of cloth to make five banners. How long could each banner be if each is the same length?"

In other cases, you need to know the remainder as a whole number. "Gina had 47 mini-bags of chocolate candies. She wanted to share them equally with four of her friends and herself. She would keep any extra bags. How many bags did each friend get? How many extra bags were there?"

(Slide 45) **Problem Based Lessons**
Have participants think about the problem for a few minutes. Then have them look at student work in handout ("How many stamps...?"). Have them talk about what each student did and compare the strategies. What are the pitfalls and/or advantages of each strategy?

Student A: Represented stamps with dots, drew square around the ones not on the border, the apparently counted them, but miscounted to get 63 rather than 64. Appropriate strategy, miscarriage of strategy.

Student B: Counted stamps on border, 10 on each side, then subtracted from the total number of 100, but didn't think not to count the corner stamps twice. So the germs of a good strategy, but miscarried.

Student C: Counted 10 stamps along top and bottom, then 8 more stamps on each side of the border, for a total of 36 on the border, subtracted from 100 which gives the correct result of 64.

Student D: This strategy counts only 9 stamps on each side which counts each corner stamp only once. It gives 36 stamps on the border with 64 left not on the border.

Student E: This strategy counts the inside stamps which would be an 8x8 square of stamps, and multiplies them to get 64.

A variety of sample problems with solutions is in handout, "Problems to Use." If you have time at any point during this module, have participants try one or more of these problems.

(Slide 46) **Avoiding Meaningless Math**
Have a volunteer read the slide.
Quoted at the Tenth International Congress of Mathematical Education in Copenhagen, 2004.

Children learn from their experiences not from what we tell them. Concepts – the ideas of mathematics – are learned through experiences and conversations about those experiences, not because they have been "told" that idea.

	(Slide 47) **Discovering Order of Operations** This activity allows children to develop the rule for order of operations for themselves. Have half the class use a calculator that is not programmed to know the rules for order of operations (like the TI-108) and the other half use a calculator that does know the rules (like the TI Math Explorer). Make a chart on a transparency or on the board to record the results. Given that, have them determine what the Explorer is doing to get its solutions and come up with the rule that multiplication and/or division are performed before addition and/or subtraction within the same expression or equation. A chart for recording results ("Discovering an Important Rule of Mathematics") is in handout. Ask participants to share their explanations for utilizing the rules of order of operations. One immediate reason may come from the fact that two different techniques provided different answers to the same equation.	
	(Slide 48) "**Good Manners of Mathematics"** After discovering the rule to multiply and/or divide before adding and/or subtracting, students need to encounter the use of parentheses. They can construct the idea that they are to pay attention to operations within parentheses first from evaluating these equations.	
	(Slide 49) **Remembering Order of Operations** Look at the triangle visual clue for remembering order of operations. Note that the E stands for exponents which are not in the elementary curriculum, but are a part of the correct order. Elementary children do not need to deal with exponents, but it may be helpful to know that exponents are in the order so that it is not a totally new idea in middle school. Ask how this visual is a better reminder of the correct order of operations than the traditional PEMDAS (Please Excuse My Dear Aunt Sally). The traditional mnemonic seems to indicate that multiplication comes before division and addition comes before subtraction, when the true order is that multiplication and division are done in the order that they appear from left to right,	

	and likewise with addition and subtraction. This can be a point of confusion for children, which the triangle visual may help to alleviate.	
	(Slide 50) **Connections** It is important that participants see how strands can be connected and embedded in each other – that connections can be made in instruction so that we are not taking more time to teach more, but can teach more by teaching better – more efficiently, more meaningfully. They should note algebra in the use of the commutative and distributive properties of multiplication, in multiplication and division equations, and in the introduction to order of operations. Have participants reflect on the activities in this module and share the problem solving that was done. What other process standards were utilized in this module? Participants should see places where they used *reasoning and proof* (justification); where they *communicated* ideas, questions, reasoning; when they made *connections* between strands of math or connections to prior knowledge; and where they used *representations* to model a situation, to help understand a problem, or to write an equation or other representation in order to solve a problem. Reiterate that the process standards should permeate mathematics instruction on a daily basis, and that they will be evident if effective instruction is going on in the classroom.	
	(Slide 51-54) **Credits and closing slides**	