Fractions as Rates: A Different Kind of Model

Adapted from Seeing Fractions: A Unit for the Upper Elementary Grades, California Department of Education, 1991

Introduction

Discuss rates with the children. Ask them what they know about rates. Consider some common rates: 3 cans of cat food for 99¢; 60 miles per hour; $3 for 2 quarts of strawberries; etc.

Give each pair of children the Fractions as Rates mat and the materials (pennies and macaroni or chips). Present this problem:

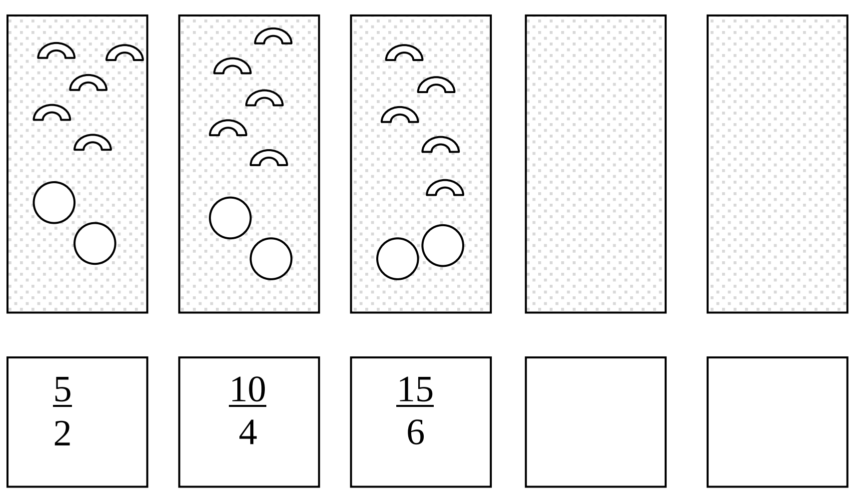
> *I bought a large box of lollipops and plan to sell them in packages of 5 candies for 2¢.* [Write the rate 5/2 on the board.*] If you want to buy Ten, how much money do you have to give me? …if you want to buy fifteen?*

Let the children think about this and show ways of solving it. Let them share their methods, making sure they explain their thinking.

Then tell them you will show them a way to generate a model of a series of rates to represent this kind of problem. Using an overhead transparency of the Fractions as Rates mat, demonstrate building rate packets – five macaronis or chips to represent the lollipops and two pennies for the 2¢. As they build, ask "After I sell the second one, what will be the total I've sold? After the third one? The fourth one? How much money will I have gotten after selling five packages?

Record the results as a number series:

The mat should look like this after three packets are represented:



Ask what would happen after you sold another packet (the sixth one). Let children share their thinking. The pose this problem:

A vending machine at the corner store dispenses seven peanuts for 4 pennies. Let's look at how many peanuts I've gotten and how much money it cost me to get them after I've used the machine 6 times. The base rate is 7peanuts for 4 pennies. [Write 7/4 on the board.]

Have the children use the mat and materials to model this rate series. When they have finished, ask them what the base rate is [7/4] and what the first five (or six or seven) terms are [14/8, 21/12, 28/16, 35/20]. Ask for explanations.

Do more rate problems with the children modeling the series to find the solution. Examples:

I decided to raise the price of my lollipops from 5 candies for 2¢ to 5 candies for 6¢. How much did I get at the first rate for 40 lollipops? How much do I get now for 40 lollipops?

Jarinda bought some new sneakers. She didn't have enough money so she had to borrow money from her big sister. She wants to pay her sister back $12 every two weeks. She borrowed $84. How long will she have to pay her sister back?

Josh wanted to add to his penny collection, so he planned to sell his two lunch cookies for 9¢ every day to another student who promised to pay in pennies. How many pennies will he have after he's sold 30 cookies?

As students are ready to create the rate series with numbers alone without the model, allow them to do so.

At this point, you may choose to have your students write some rate problems. You can suggest rates (2/3; 4/7; 3/8; etc.) or they could come up with their own. You can type up their problems so they the students can try each other's problems.

Comparison Problems Using the Rate Series

Allow students to continue using the materials to model the rate series as long as they need to.

Present this problem to the class:

> *At the fair last weekend, I bought gumballs from a machine that dispensed 3 Gumballs for 8 pennies. My friend John found a different machine that gave her 12 gumballs for twenty cents. Find a way to compare them to find out who got the better deal?*

After students have had a chance to work on the problem, record the two series on the board. (Let them tell you the numbers to record.)

My machine:	$\frac{3}{8}$	$\frac{6}{16}$	$\frac{9}{24}$	$\frac{12}{32}$	$\frac{15}{40}$	. . .
John's machine:	$\frac{12}{20}$	$\frac{24}{40}$	$\frac{36}{60}$	. . .		

When you have fractions with a common element, you can compare them to see which is a better deal. For example, 15 for 40 and 24 for 40. With John's machine you get more gumballs for the same amount of money. Another comparison can be made at 12 for 32 and 12 for 20. In this case, you get the same amount of gumballs, but you spend less money in John's machine. Another comparison that some children see is 9 for 24 and 12 for 20. My machine gives less gumballs for more money that John's machine which gives more gumballs for less money. [Note that the comparisons are not necessarily made in the same term in both series.]

Have the children solve other comparison problems. They should not only show the rate series, but should explain their conclusion in words; e.g., John's machine is a better deal because you get more gumballs for less money.

Sample comparison problems:

> *Ricky sells Pokemon cards at 3 for 35₵. Is that a better deal than Sarah selling them at 4 for 50¢?*
>
> *Rakene bought a giant bag of Jolly Ranchers and sells them at 4 for 5₵. His friend Jake found a machine that dispensed them at 9 for 25₵. Which is a better deal for you?*

The children can also write comparison problems for each other to solve.

Patterns in Rate Series

Have the children generate the rate series for the base rate of 4/5. Write it on the board up to the fifth term.

$$\frac{4}{5} \quad \frac{8}{10} \quad \frac{12}{15} \quad \frac{16}{20} \quad \frac{20}{25}$$

Have the children look for patterns in the series. List them and have the class check them to be sure they work as the series is extended. Is the pattern useful for predicting the sixth term? the tenth term? etc.

Patterns they might see include:

a) The top numbers increase by 4 each term; are all multiples of 4.
b) The bottom numbers increase by 5 each term; are all multiples of 5.
c) The sum of the top and bottom numbers increase by 9 each term: 4+5=9, 8+10=18, 12+15=27, etc.
d) The difference between the top and bottom numbers increase by 1 each term: 5-4=1, 10-8=2, 15-12=3, etc.
e) To get the fourth term, multiply the numbers in the base term by 4; to get the sixth term, multiply the numbers in the base rate by 6, etc.
f) The increase from the top number of one term to the bottom number of the next term increases by one more for each term: 4+***6***=10, 8+***7***=15, 12+***8***=20, etc.

After a class discussion of the patterns in the rate series for 4/5, have the children work in small groups to find patterns in other rate series. Give each group a different series. Examples: 2/3, 4/9, 5/7 3/10, etc. After they have had time to generate a list of patterns, have them share with the whole group. List the patterns on a transparency or on chart paper. Look at the patterns to see if they are generalizable to other base rates. For example, pattern e above is generalizable to any base rate. Other base rates will have similar patterns to a, b, c, d, and f, but the numbers may be different.

Caution: As the children build rate series without the benefit of the materials, some children will build a series like this one:

$$\frac{2}{7} \quad \frac{4}{14} \quad \frac{8}{21} \quad \frac{16}{28}$$

in which the numbers in each term are doubled from the term before. Allow these children to continue using the materials as long as necessary.

Finding Equivalent Fractions in a Rate Series

As the children generate rate series, ask them to predict the sixth term, tenth term, eighth term, and then build the series out with numbers (and with materials, if needed). Ask them how they are predicting. Take the rate series 4/5:

$$\frac{4}{5} \quad \frac{8}{10} \quad \frac{12}{15} \quad \frac{16}{20} \quad \frac{20}{25}$$

To predict the sixth term, children generally simply add another 4 and another 5 to the numbers in the fifth term. For the tenth term, many children double the numbers in the fifth term:

$$\frac{20 \times 2}{25 \times 2} = \frac{40}{50}$$

Others will see that the numbers in the tenth term are ten times the numbers in the base rate:

$$\frac{4 \times 10}{5 \times 10} = \frac{40}{50}$$

Note that this is the conventional algorithm for finding equivalent fractions. As the children make these discoveries, they are constructing the meaning of this algorithm for themselves.

Caution: Be careful as you record these observations, that you are not leading the children to believe that they are multiplying 4/5 by 10 or 20/25 by 2. Emphasize that as they multiply both the top and bottom numbers by the same number they are really multiplying by one. 2/2 = 1; 10/10 = 1, etc. Ask them what happens when you multiply a number by one and let them tell you that you get the same amount. Emphasize that as they use this pattern to find numbers in the rate series, they are changing the fraction name, but not its value. In each case, you still have the relationship of the base rate.

More problems:
Present this problem to the class:

> *My neighbor went out of town and asked me to feel her cat. I earn two dollars every time I feed the cat three times. I fed the cat twelve times the last time she was out of town. How much did I earn?*

Write the rate on the board: $\frac{2}{3} = \frac{?}{12}$

Have the children find ways to solve. Some children will see that the 12 is the fourth term out (because 3 × 4 is 12) and will know that the ? will be four 2's. Others will need to build out the rate series with numbers. Some may still need to use the materials, but most if not all will be able to use the numbers.

Then ask questions like these:

Can you find the seventh term of the rate series with base rate ¾?

Use your best method. How did you think about it? How did you do it?

Find the fifth term of the rate series based on 1/7.

Find the ninth term of the series with base rate 4/9.

Make up other questions like these to give the children lots of opportunity to internalize and make meaning for themselves of this method for finding equivalent fractions.

Finding Equivalent Fractions in a Rate Series: Simplifying the Numbers
The preceding experiences lead children to a method for finding equivalent fractions when the numbers in the fraction get larger. These problems will help them learn ways to find equivalent fractions when the numbers get smaller.

Present this problem:

Connor bought 12 stickers for his collection for 40¢. Jamal wanted to buy some stickers too, but he didn't have 40¢. What would be the least amount of money he could spend for stickers at that rate?

Make sure materials are available for students who need them. Let students work on the problem, then record their strategies on the board.
For this problem, students have to work backwards. They might think like this:

$\frac{12}{40}$ Split both numbers in half to make: $\frac{6}{20}$ and again: $\frac{3}{10}$

The 3 and 10 can't be divided again by the same amount, so the least amount of money Jamal could spend would be 10¢.
The algorithm being used is this: $\frac{12 \div 2}{40 \div 2} = \frac{6}{20}$

Some children might see that both 12 and 40 are divisible by 4, and might go directly to the solution: $\frac{12 \div 4}{40 \div 4} = \frac{3}{10}$

Record the students' strategies, using this notation when appropriate.

Present another problem:

Jolene is having a birthday party. The party favors she wants are priced at 12 for 20¢. She needs 35 favors, and the shopkeeper is willing to split a package for her. How much will the 35 favors cost?

Write on the board: $\frac{12}{20} = \frac{?}{35}$

Let the children grapple with this problem, then give them an opportunity to share their strategies and solutions.

Some children will see that you can go backwards to a simpler rate by dividing 12 and 20 by 4 to get a base rate of 3/5, and then find the seventh term of that base rate, which is 21/35. The favors will cost 21¢. Use appropriate notation to record the children's strategies and thinking.

Fractions as Rates: Generating a rate series with materials and numbers

Use pennies and another material (macaroni, chips, etc.)

Rates (Use numbers.)