Fraction Computation

Use a problem-based number sense approach

- Begin with simple tasks in contexts.
- Connect the meaning of fraction computation with whole-number computation.
- Develop strategies using estimation and informal methods.

Use models to explore each of the operations.

Addition and Subtraction

Begin with informal exploration of simple contextual problems.

Carla and Geoff were each eating the same kind of candy bar. Carla had ¾ of her bar left. Geoff still had 7/8 of his bar left. How much candy did the two children have together?

Using nothing other than simple drawings, how would you solve this problem without setting it up in the usual manner and finding common denominators?
Try to think of two different methods.

Sometimes physical models work better than drawings.

Marie and Glen ordered two identical-sized pizzas, one pepperoni and one veggie. Marie ate 5/6 of a pizza and Glen ate ½ of a pizza. How much pizza did they eat altogether?

What model would you use for the whole? (fraction strips – Cuisenaire rods; fraction bars, fraction circles…?)
How would you solve this problem without a traditional algorithm?

Most of the problems you pose should involve simple fractions with friendly denominators no greater than 12. At the same time, do not be afraid of including mixed numbers and unlike denominators.

Build on informal explorations and invented strategies to develop an algorithm for addition and subtraction and to see that the common-denominator approach is meaningful.

Like denominators:
If students have a solid understanding of fraction concepts, they should be able to add and subtract fractions with like denominators (e.g., 2/5 + 4/5, or 4 7/8 – 2 3/8) right away. Understanding that the numerator is the count and the denominator is what is counted makes adding and subtracting like fractions the same process as adding whole numbers. Two fifths + 5 fifths is the same problem as adding two apples and four apples. If children have problems with adding and subtracting like fractions, they need further work with fraction concept development before moving to adding and subtracting unlike denominators.

Unlike denominators:
Consider a problem where only one fraction needs to be changed:

$$\frac{2}{4} + \frac{5}{8}$$

- Use models.
- Convert the problem. Ask the key question: "How can we change this to a problem with the parts the same?" - like "adding apples and apples". (In this case fourths can be changed to eighths.)
- Main idea: 2/4 + 5/8 is the same problem as 4/8 + 5/8

Consider a problem where both fractions need to be changed:

$$\frac{2}{3} + \frac{1}{4}$$

- Focus attention on ***rewriting the problem*** in a form that is "like adding apples and apples" so that the parts of both fractions are the same.
- Be sure children understand: The new form is the same problem as the old form.
- Continue to have children demonstrate with models.

Subtraction of two simple fractions follows the same approach.

A separate algorithm for adding and subtracting mixed numbers is not necessary.
Include mixed numbers in all fraction addition and subtraction activities.
Let students solve these problems in ways that make sense to them.
Note that students tend to work with the whole numbers first.

Developing the need for a common denominator or common family name for two or more fractions with different denominators

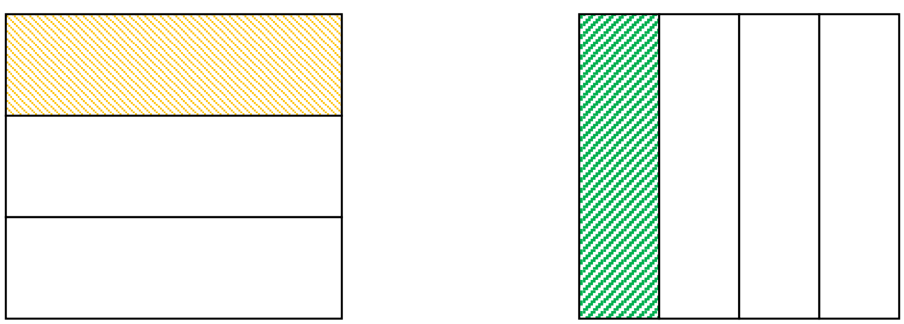
Using two different shapes or items on the overhead projector, ask students to add the two groups and decide what to call the answer. For example, show two cats and three dogs and ask the class to add them. When they answer, "five," ask "five what? What name can we give to both groups? To what family do they both belong?" Students are likely to say "animals." Verbalize the response: "Two cats and three dogs equal five animals." Continue with other examples, such as:

5 black cats + 3 brown cats = 8 cats
14 triangles + 17 squares = 31 polygons
9 apples + 7 oranges = 16 pieces of fruit
25 owls + 13 robins = 38 birds

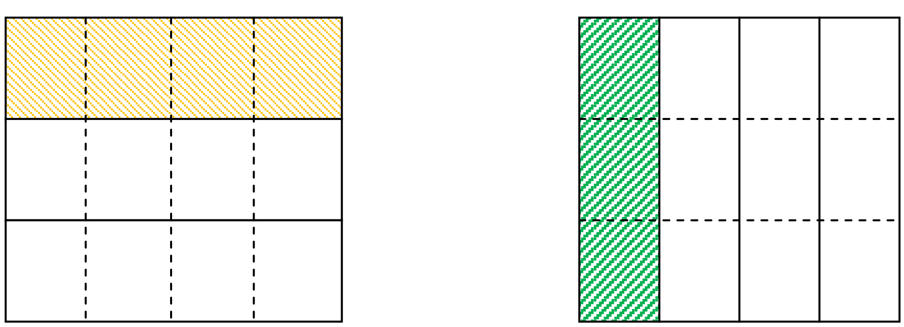
Make the point that you are looking for a common attribute – a common name for the two groups – in order to name the result.

After several examples of finding a common name, give students the recording sheet "Fraction Squares." Using an overhead transparency of two squares, model showing two different fractions on adjacent squares like this:

$\frac{1}{3}$ $\frac{1}{4}$



Shade in the appropriate part of each square. Then draw dotted lines in the opposite squares to create pieces which can be counted in a common way in both squares, like this:



Be sure the children note the following:
There are 12 pieces total (in one whole square).
1/3 = 4/12 (4 of the 12 squares in a whole).
1/4 = 3/12 (3 of the 12 squares in a whole).

So 1/3 + ¼ = 4/12 + 3/12 = 7/12.

Twelfths is a common name for thirds and fourths.

Do several examples like the previous one with the students, using the Fraction Squares recording sheet. Possible problems include:

1/5 + 2/3	1/3 + 4/7
1/8 + 1/3	1/6 + ¾
2/7 + ½	2/5 + ¾(answer will be improper)

****Note*: The language you use as you discuss the addition and subtraction of fractions is very important. For instance, when adding ¼ + ⅓ , you should say, "How many?," a counting question, rather than "How much?," a unit question. ¼ and 3/12 are the same amount (how much), but ¼ and 3/12 are not the same number of pieces (how many). To add two numbers, fractions or otherwise, you must figure out a way to count the "<u>how many</u>." If the pieces are already the same size, they may be counted as they are. If they are not the same size, you must find a common size piece that fits both pieces you want to add. (The denominator is like a last name. It tells which fraction family you are talking about. A common denominator is simply the family into which both fractions fit.)

Finding the Least Common Denominator – One Method

A common denominator is also a common multiple of the denominator numbers. In this part the children find the smallest common denominator for a pair of fractions by listing the multiples of each denominator until they find the least common one.

In the previous activity, using the Fraction Squares to find a common family name for two fractions, the common denominator was the product of the two denominators. As the children do the activities in this part, point out to them that sometimes the least common denominator is one of the denominators already given, sometimes it is the product of the two denominators, and sometimes it is a number less than the product of the two denominators which is a common multiple of both denominators.

Do a few examples on the board like the following:

To find the least common denominator (LCD) of 4/7 and 2/5, list the multiples of 7 and the multiples of 5:

Multiples of 7	Multiples of 5
7	5
14	10
21	15
28	20
35	25
42	30
49	35

35 is the smallest common denominator for 3/7 and 1/5.

$$\frac{3}{7} = \frac{15}{35} \qquad \frac{1}{5} = \frac{7}{35}$$

Note that in this case, the least common denominator is 35, the product of the two denominators 7 and 5.

To find the least common denominator of 3/8 and 7/10, list the multiples of 8 and 10 until there is a common one:

Multiples of 8	Multiples of 10
8	10
16	20
24	30
32	40
40	50

In this case the least common denominator is not the product of the denominators 8 and 10, but is 40. 80 *is* a common denominator of 3/8 and 7/10 and can be used to solve a problem involving 3/8 and 7/10 or to compare and order them, but using 40, the least common denominator, will provide a solution that is simpler (in lower terms) that may be easier for the children to visualize.

Use the following problems for more class discussion if necessary:

Gregory bought 2 2/3 kg of heel powder on Thursday. By Monday, he only had 1 4/9 kg left. How much heel powder had he used?

(one denominator changes)

2 2/3 – 1 4/9 = ☐

2 6/9 – 1 4/9 = ☐

Solution: 1 2/9 kg

Helene bought some catnip powder. After she had used only 1/6 kg of it, her cat bothered her so much that she threw away the rest, all 1 ¾ kg! How much catnip powder did she buy?

(both denominators change)

1 ¾ + 1/6 = ☐

1 9/12 + 2/12 = ☐
or:
1 18/24 + 4/24 = ☐

Solution: 1 11/12 kg or 1 22/24 kg

Least Common Multiples Flash Cards

(from Van de Walle and Lovin, Teaching Student-Centered Mathematics, Grades 3-5)

Make flash cards with pairs of numbers that are potential denominators. Most should be less than 16. For each card, students try to give the least common multiple (LCM). Be sure to include pairs that are prime, such as 9 and 5; pairs in which one is a multiple of the other, such as 2 and 8; and pairs that have a common divisor, such as 8 and 12.
Examples:

3, 5 → 15   4, 8 → 8   6, 8 → 24

Fraction Computation Practice

The best practice is within the context of a problem to solve.

A.
After his birthday party, Yoshi had 5/8 of his birthday cake left. He and his sister ate another 2/8 of the cake and drank ½ of the fruit juice that was left. What fractional part of the cake was left then?

B.
Toni likes to compete in the standing broad jump. Her jumps at Saturday's meet were 3 7/8 yards, 2 1/2 yards, 1 2/8 yards, and 2 ¼ yards. What was the difference between her longest and shortest jumps?

C.
Jess walked 2 ¼ miles to the store. When he left the store he walked 1 2/5 miles to his friend's house. Then he walked home. If he walked 5 ¾ miles altogether, how many miles is it from his friend's home to Jess's home?

D.
Kari spent 3 1/3 hours on Saturday selling raffle tickets for her school and she sold 60 tickets. She spent another 2 ¼ hours selling tickets on Monday afternoon and sold another 45 tickets. How many hours did she spend selling tickets altogether?

E.
Carmen's Aunt Matilda always sent Carmen a box full of paperback books for her birthday. By the weekend after her birthday, Carmen had read 3/7 of the books in the box. By the next weekend, she had read another 2/5 of the books. What fractional part of the books had she not yet read?

F.
Sylvia's teacher asked her to solve this problem:

4/12 + 1/3 = ?

Sylvia got the answer 5/15.
Was she correct?

Use pictures, models, or words to show or tell why she was right or wrong.