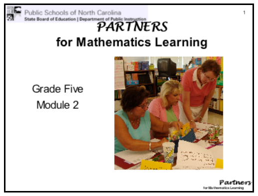
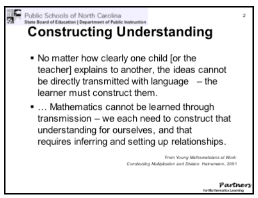
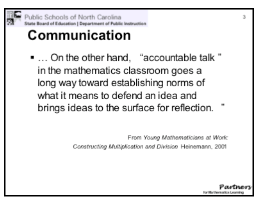
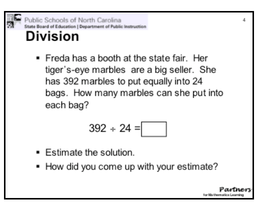
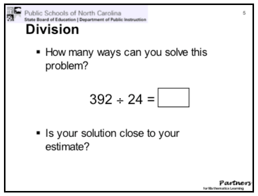
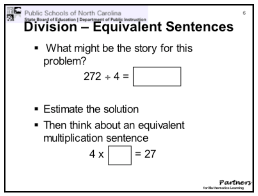
General Materials and Supplies:
Calculators, some programmed for order of operations, e.g., TI Math Explorer, and some not, e.g., TI-108

<table>
<tr><th>Slide</th><th>Tasks/Activity</th><th>Personal Notes</th></tr>
<tr><td>
</td><td>(Slide 1) Partners Grade 5 Module 2
Number and Operations;
This is Part 2 of the Number and Operation strand. This module will focus on multiplication and division of whole numbers and of fractions.</td><td></td></tr>
<tr><td>
</td><td>(Slide 2) Constructing Understanding
This is another way to say that children construct their own understanding through experiences and conversations. We must design instruction to give children these kinds of opportunities.</td><td></td></tr>
<tr><td>
</td><td>(Slide 3) Communication
The conversations that take place in math class between students and teachers and student-to-student help children explore and explain their own thinking and learn how to justify their own conclusions.</td><td></td></tr>
<tr><td>
</td><td>(Slide 4) Division
Ask participants to estimate a solution. Then have them share how they arrived at the estimate. One possibility is to think about 392 as close to 400 and 24 as close to 25. Since there are 4 sets of 25 in 100, there would be 4x4 sets of 25 in 400, so 16 is a reasonable estimate.</td><td></td></tr>
</table>

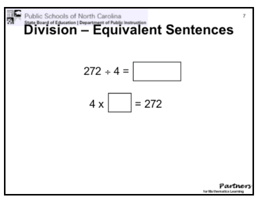


(Slide 5) **Division**
Ask participants to find ways to solve this problem without the traditional algorithm.

Use the "Math Talk" directions for a whole group discussion; or share at tables first, then with the whole group.

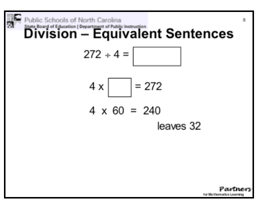


(Slide 6) **Division – Equivalent Sentences**
Providing a context for computation helps to talk about the process of solving. Rather than always giving the context yourself, allow children to give you the story for the problem. That also gives you the opportunity to assess their understanding of the operation, in this case division.
Go to the next slide for a possible context and one solution method.



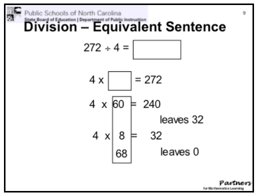
(Slide 7) **Division – Equivalent Sentences**
Show this method step by step. Giving it a context helps with the conversation, whether you provide the context or have children give you the story for the problem. Suppose this one is "The PE teacher has 272 foam balls that need to be put into 4 bins with an equal number in each bin. How many balls will go in each bin?"

First ask participants to write an equivalent multiplication sentence to help think about the solution. (For example, "4 bins, each containing ___ balls, will have a total of 272 balls.")



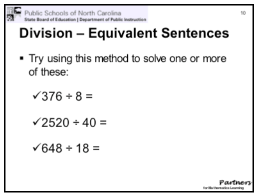
(Slide 8) **Division – Equivalent Sentences**
Using the multiplication sentence as a guide, begin to find multiples of 10 (or 100, depending on the size of the numbers involved) that could go into each bin. In this case there are enough balls for 60 groups of 4 or 240 balls to go into each bin. That takes care of 60 x 4 or 240 balls, which leaves 32 balls to go into the bins.

With this method, if the children try a smaller number of balls than 60, they will have more than 32 balls left, but can simply put in another multiple of 10 until 32 balls are left.



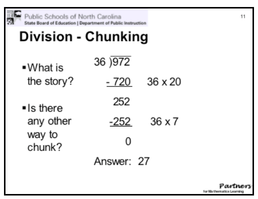
(slide 9) **Division – Equivalent Sentences**
Since 4 x 8 is 32, 8 more balls can go into each bin, making 60 + 8 or 68 balls for each bin.

The process is so much more meaningful and easy to understand when put into a context and talked through in terms of that context.



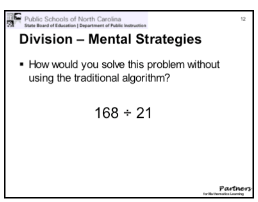
(Slide 10) **Division – Equivalent Sentences**
Have participants try the equivalent sentences method to solve one or more of these division problems.
(376 ÷ 8 = 47; 2520 ÷ 40 = 63; 648 ÷ 18 = 36)

See handout, "One Alternate Strategy for Division: Missing Factor".

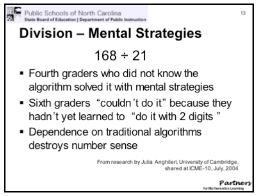


(Slide 11) **Division - Chunking**
This is a different notation, but the process is similar. Again, provide or ask for a story to give the problem a context. In this case, in the first step, 20 groups of 36 are taken care of first, leaving 252 which is 7 groups of 36. Again, the child could try 10 groups of 36 first, and then take another 10 groups of 36.

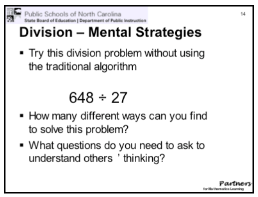
Be sure to talk about the problem in terms of the story. Also ask participants if they would've "chunked" in a different way (Ex. Chunking by 10 twice in the beginning). Then have them try 828 ÷ 23. (Answer is 36.)



(Slide 12) **Division – Mental Strategies**
Have participants try this problem using mental strategies rather than the traditional algorithm. Have them share their strategies. Then show the next slide.

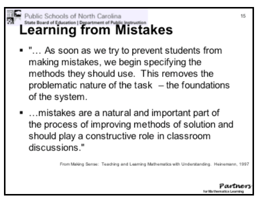


(Slide 13) **Division – Mental Strategies**
This is another example of students becoming so tied to traditional algorithms that they "can't" solve a problem even when it lends itself to mental strategies. It is imperative that we delay teaching traditional algorithms long enough for children to become comfortable with their own strategies, realizing that with the algorithm and their strategies, they have an arsenal for solving any problem.



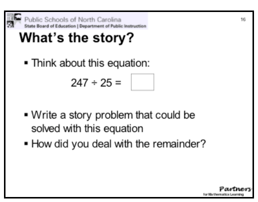
(Slide 14) **Division – Mental Strategies**
Have participants explore various strategies for solving this problem without using the traditional algorithm. Allow time for them to share their ideas with the whole group.

As participants share strategies, ask the others to think about what questions they would need to ask their students in order to understand their thinking.



(Slide 15) **Learning From Mistakes**
Mistakes can be springboards toward understanding if treated appropriately. Share the story that Edison made many attempts during 1½ years of work to perfect a practical, economical, safe incandescent light bulb before he was successful. From each attempt that didn't work, he could learn something to help him try a different approach. We should allow our children the same opportunities.

Mistakes should be evaluated to see what is wrong and how it can be "fixed" rather than as responses to receive a big red X with no feedback or discussion.



(Slide 16) **What's the Story?**
Use this slide to talk about interpreting remainders. Have participants work together in table groups to write a problem could be solved using the given equation. Have them share their problems and their solutions, specifically addressing how they dealt with the remainder.

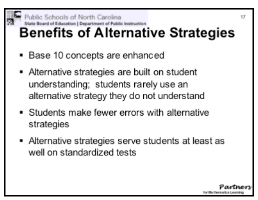
Discuss that deciding what to do with the remainder depends on the context of the problem. Be sure that the following possibilities for interpreting the remainder are discussed. If the participants' problems don't address one or more of these possibilities, be sure to include these in the conversation.

1) Some kinds of problems require that the remainder be ignored. "Johnny had 247 stamps

to add to his collection. If he put 25 on each page, how many full pages would he have?"

2) Others require a solution that adds one to the whole number quotient. "Shana had 247 marbles. She made cloth bags to hold the marbles. If she put all the marbles in bags with 25 marbles in each bag, how many bags would she need so that all the marbles were in a bag?"
3) In other cases, the quotient can be expressed as a mixed number. Jordan had 247 feet of cloth to make five banners. How long could each banner be if each is the same length?"
4) In other cases, you need to know the remainder as a whole number. "Gina found a great bargain for mini-bags of chocolate candies. She bought 247 mini-bags. She wanted to share them equally with four of her friends and herself. She would keep any extra bags. How many bags did each friend get? How many extra bags were there?"

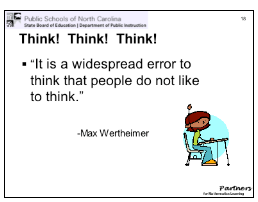
See Handout, "Interpreting Remainders," for examples of problems for which decisions must be made about how to use the remainders.



(Slide 17) **Benefits of Alternative Strategies**

Have participants read these points.

Emphasize the last point that research is showing that students who use their own invented strategies do at least as well on standardized tests as students who rely on traditional algorithms. Could this be because they understand what they are doing?

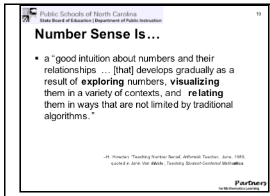


(Slide 18) **Think! Think! Think!**

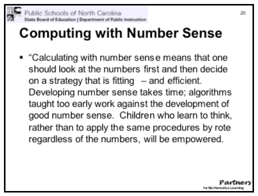
Students enjoy the opportunity to think and express themselves when they are allowed to try methods and ideas without threat of being put down for mistakes, rather than using mistakes as an opportunity to learn.

Using rubrics to score problems allows children to receive good feedback for their efforts rather than just a "grade."

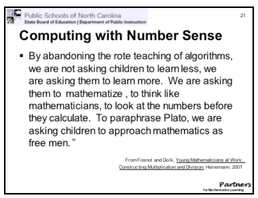
(Slide 19) **Number Sense is…**



We've been utilizing our number sense. Here is another good definition of number sense that points out the activities that lead to the development of number sense.

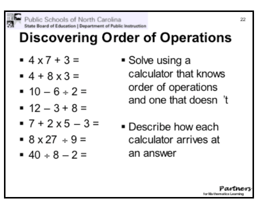


(Slide 20) **Computing with Number Sense**
This quote, completed on the next slide, is from *Making Sense*, Heinemann, 1997. It underscores the need to help children develop strategies derived from the numbers they are working with, not just looking at single digits or following rote procedures.



(Slide 21) **Computing with Number Sense**
This slide completes the quote. We must ask our children to think, reason, and solve problems, if we are to prepare them for the unknowns of this 21^{st} century. They will have problems to solve that we cannot even imagine. We must not cheat them out of the extraordinary opportunity that mathematics presents to develop the skills they will need to solve those problems.

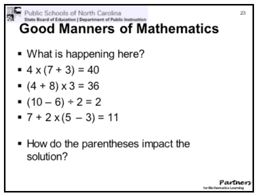
Memorizing facts and procedures without meaning will not "do the trick" to prepare them to meet these challenges. Thinking through problems, working collectively, evaluating and justifying solutions, learning to take risks and learn from mistakes – all these will help children be ready for whatever they may face in the future. We have to be willing to take the risks involved in helping children develop these thinking and risk-taking skills.



(Slide 22) **Discovering Order of Operations**
This activity allows children to develop the rule for order of operations for themselves. If children have not done this in fourth grade, they will need this experience in fifth.

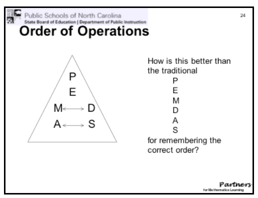
Have half the class use a calculator that is not programmed to know the rules for order of operations (like the TI-108) and the other half use a calculator that does know the rules (like the TI Math Explorer).

Make a chart on a transparency or on the board to record the results. When all results are recorded, tell them that the Explorer is programmed to know the rule for the order in which operations should be done but the other calculator does not. Given that, have them determine what the Explorer is doing to get its solutions and come up with the rule that multiplication and/or division are performed before addition and/or subtraction within the same expression or equation. See the next slide for the next step.



(Slide 23) **Good Manners of Mathematics**

After discovering the rule to multiply and/or divide before adding and/or subtracting, students need to encounter the use of parentheses. They can construct the idea that they are to pay attention to operations within parentheses first from evaluating these equations.



(Slide 24) **Order of Operations**

Look at the triangle visual clue for remembering order of operations. This can be a review if children have seen this in fourth grade. Note that the E stands for exponents, which are not in the elementary curriculum, but are a part of the correct order. Elementary children do not need to deal with exponents, but it may be helpful to know that exponents are in the order so that it is not a totally new idea in middle school.

Ask how this visual is a better reminder of the correct order of operations than the traditional PEMDAS (Please excuse my dear Aunt Sally). The traditional mnemonic seems to indicate that multiplication comes before division and addition comes before subtraction, when the true order is that multiplication and division are done in the order that they appear from left to right, and likewise with addition and subtraction. This can be a point of confusion for children, which the triangle visual may help to alleviate.

Ask participants why these rules of order of operations are important.

(Slide 25) **Order of Operations**

	5th graders should begin to be able to apply order of operations in meaningful contexts. This activity asks participants to write equations to match a story problem. See the next slide for one solution and discussion. This problem requires a knowledge of order of operations to write and solve the equation.	
	(Slide 26) **Order of Operations** Did participants write any other equations? Discuss the reasons order of operations is important in this problem. (Order of operations is part of the language of mathematics – the rules for recording mathematical ideas – that allows us to understand the work of another person.) See handout, "Using Order of Operations to Write Equations" for other examples of problems that could be represented by equations requiring order of operations to solve. The next slide asks participants to write story problems to match equations.	
	(Slide 27) **Order of Operations** This activity asks participants to come up with a story problem to match each equation. Assign one example to each table and allow them to work on others if there is time. Have tables share their stories and discuss how the story matches the equation and why the order of operations is important in finding the correct solution to the story problem.	
	(Slide 28) **Avoiding Meaningless Math** Quoted at the International Congress of Mathematical Education in Copenhagen, 2004. Children learn from their experiences not from what we tell them. Concepts – the ideas of mathematics – are learned through experiences and conversations about those experiences, not because they have been "told" that idea.	
	(Slide 29) **Big Ideas in Fractions**	

This slide begins a look at fractions in Grade 5. Briefly look at each bullet. Each will be elaborated.

The second bullet means the names for fractional parts – thirds, fourths, etc. The fourth bullet is about the compensatory principle.

(Slide 30) **Fractions**
These three bullets express meanings of fractions.

Values: Fractions express parts of a whole and can be represented on a number line. They can be ordered and operated on (added, subtracted, …). They can be counted in sequence, just as whole numbers can (e.g., 1/3, 2/3, 3/3, 4/3, …). Remind participants of the counting by fractional parts activities that were done in Partners year 1.

Operators: One can find a fractional part of a value (1/4 of 12, for example). The fraction symbol can be an expression of a division problem. 12/4 is a symbolic representation of 12 divided by 4. It is important that children begin to see this meaning of a fraction as a division problem, a common representation of division in algebra.

Ratios: A ratio is a comparison of two quantities. For example 15/3 could represent 15 gumballs shared equally by 3 people (15 to 3). Likewise 3/15 can represent 3 pizzas shared by 15 people (3 to 15).

(Slide 31) **Fraction Actions**
These are applications of the big ideas of previous slide. Ask participants which applications involve which meanings of fractions.

(Slide 32) **Understanding Fractions**

	To help children truly understand fractions, they need experiences beyond dividing one whole into fractional parts. Experiences like these help them take a known part of a whole and create the whole, deepening their knowledge and understanding of fractions. Handouts "What's the Whole?" and "Fractions of Words I and II," provide samples of activities like these. Ask participants to come up with solutions. Possible solutions: Bullet 1: The whole could be a trapezoid and 2 triangles or a trapezoid and1 rhombus. Bullet 2: There are lots of possibilities: milk (1 vowel out of 4), inchworm (2 vowels out of 8) Bullet 3: The yellow Cuisenaire rod is 5/8 of the brown rod. Bullet 4: 12 (4 children per group x 3 groups)	
	(Slide 33) **Fraction Computation** Discuss each bullet briefly. Note that instruction in computation with fractions begins with contextual problems and the use of informal methods and models to solve.	
	(Slide 34) **Fraction Addition and Subtraction** Have participants work together to come up with two different ways to solve this without the algorithm. As they share, make sure that their methods make mathematical sense.	
	(Slide 35) **Fraction Addition and Subtraction** Again, have participants solve the problem without using a traditional algorithm and share solution methods. Discuss what models they suggest using for the whole. What models would be the best representations?	
	(Slide 36) **Fraction Addition and Subtraction**	

	Talk through these points. When we push children who aren't conceptually ready, we give them procedures to follow without understanding.	
	(Slide 37) **Fraction Addition and Subtraction** Have participants estimate solutions. For the first problem they should see that it would be slightly less than 2. The second problem would be less than 1, but more than 2/3. The third problem would be exactly 1, but an estimate of "about 1" would be reasonable. Ask participants how they came up with their estimates. Discuss the two questions.	
	(Slide 38) **Fraction computation** Have participants work together at tables, and then share some strategies. Ask: What models might you use? Could you use a number line?	
	(Slide 39) **Common Denominators** This will be very different from the instruction that teachers are used to giving related to computing with fractions. The result will be an algorithm that makes sense to kids rather than a procedure to be followed, which often results in mistakes.	
	(Slide 40) **Fraction Computation** The big idea here is to convert the problem so that it is like adding apples and apples. Ask participants to expand upon what they think is meant by "same parts".	
	(Slide 41) **Fraction Computation**	

Children need to see that the new form of writing the problem is the same as the old.

How would models help children see that the problem has not changed – only the form of the problem has changed, so that the denominators are the same (adding "apples to apples")?

(Slide 42) **Equivalent Fractions**
Make the point that children do need a method for finding equivalent fractions, but it should be a method they understand and have developed from problem-based explorations, which allow them to use their own intuitive methods to find equivalent fractions. "Rules" should come out of these explorations and be based on the children's experiences.

(Slide 43) **Equivalent Fractions: Developing the Concept**
Tell participants that you are going to introduce them to an excellent model for the construction of the algorithm for finding equivalent fractions. While students should have learned to do this in 4th grade, this is an excellent tool for teaching understanding, for those students who need it.

This model is from California's Beyond Activities Project's Seeing Fractions unit, published in 1991. See handout pages (beginning with Fractions as Rates) for instructional directions.

Use the introduction in the first page of this section of handouts to begin this exploration. Participants will use objects to create a concrete model of the rate series on the workmat, handout (last page in the Fractions as Rates set of handouts).

After a couple of examples using the materials, have them begin to represent the rate series in each problem symbolically. Point out that after this experience, children can write rate problems for each other to solve.

Then using the guide in this section of handouts, to introduce the participants to using the rate series in comparison problems. Do one or two of these with the group. Be sure that participants see the various ways that rates may be compared.

(Slide 44) **Equivalent Fractions: Developing the Concept**

Continue to use the Rate Series instructional guide in the "Fractions as Rates" handouts as your guide.

The third section, Patterns in the Rate Series, is a rich activity in that there are many patterns they may see, hopefully including the one that they can find a fraction with the same value as a given fraction by multiplying both numerator and denominator by the same number.

The fourth section, Finding Equivalent Fractions in a Rate Series, solidifies this understanding, as children generalize from the pattern they saw in earlier experiences.

Point out how important it is that the notation teachers use is mathematically correct.
Continue with using the rate series to simplify numbers by dividing both parts of the fraction by the same factor. See handouts for examples to use.

Point out the connections across strands that are built into these rate activities. The rate series not only leads children to construct the algorithm for finding equivalent fractions and gives it meaning, but these experiences also give children an opportunity to use algebraic thinking and pattern recognition. This model is a ratio example of fractions, and is essentially a set of ordered pairs, which could be graphed on a coordinate grid.

(Slide 45**) Fraction Computation**
Moving back into the application of equivalent fractions in solving addition and subtraction problems, look at the problems on the slide.

In the usual algorithm, the thirds would be changed to sixths (the smaller denominator to the larger), but in this example, the sixths could also be changed to thirds. Discuss this possibility. Mathematically, it is certainly accurate, so children should be allowed to use this strategy. But as a warning, look at ¼ + 7/8. In this case the eighths cannot be changed to fourths (without creating a complex fraction), so the fourths will need to be changed to eighths.

What we are talking about is helping children develop the fraction number sense to be able to change the fractions appropriately (in either direction).

(Slide 46) **Common Denominator**

Let participants read the quote on the slide.

The handout, "Fraction Computation," provides a discussion of and strategies for instruction in addition and subtraction involving fractions and in finding common denominators and least common denominators.

(Slide 47) **Mixed Numbers**

This is contrary to the way most textbooks introduce addition and subtraction of fractions, but if we are taking a number sense approach, it makes sense to include mixed numbers in addition and subtraction activities.

Make note that students tend to pay attention to the whole numbers first, in much the same way that they work from left to right in whole number computation. For example, if they add 2 2/3 and 4 4/9, they will add the 2 and 4 to get 6, then add the 2/3 and 4/9 to get 12/9 or 1 3/9 or 1 1/3. They can easily add one more to the 6, to make a sum of 7 3/9 or 7 1/3.

The handout, "Fraction Computation Practice," provides sample problems for adding and subtracting fractions.

(Slide 48) **Balancing Fractions**

The balance format brings algebraic thinking into the work with number, and helps to develop meaning for fractions and operations with fractions. When using this format in working with fractions, students are more likely to develop solution strategies that are meaningful to them, rather than following a formula.

In the top balance, x = 3 ½. Participants may think:

1 + 1 = 2;

¾ + ¾ = ½ + ½ + ¼ + ¼ = ½ + ½ + ½ = 1 ½;

2 + 1 ½ = 3 ½ .

In the second balance, since 1 = ½ + ½ , the block 1 ½ = 3 sets of ½ so each of the three cylinders, "n" = ½ .

Before leaving this slide, be sure that participants see that activities like this involve both algebra and number/operations so that more than one strand can be addressed at the same time.

(Slide 49) **Deep Mathematics**

	This is a quote from Cathy Seeley, a past president of NCTM. Go to the next slide for the rest of the quote.	
	(Slide 50) **Deep Mathematics** Make the point again that rules must follow an understanding of the numbers involved (in this case fractions) and of the operations being used. Point participants to the article by Seeley on the CD. It is an excellent one to share with parents and other interested parties.	
	(Slide 51) **Connections** It is important that participants see how strands can be connected and embedded in each other – that connections can be made in instruction so that we are not taking more time to teach more, but can teach more by teaching better – more efficiently, more meaningfully. They should see algebra in the equations adding and subtracting fractions and in the balance problems which informally work with systems of equations and variables. Also, non-standard strategies often rely on operation properties (commutativity, associativity, and the distributive property), items seen in the algebra strand. Have participants reflect on the activities in this module and share the problem solving that was done. What other process standards were utilized in this module? Participants should see places where they used reasoning and proof (justification); where they communicated ideas, questions, reasoning; when they made connections between strands of math or connections to prior knowledge; and where they used representations to model a situation, to help understand a problem, or to write an equation or other representation in order to solve a problem. Reiterate that the process standards should permeate mathematics instruction on a daily basis, and that they will be evident if effective instruction is going on in the classroom.	
	(Slide 52-55) **Credits and closing slides.**	