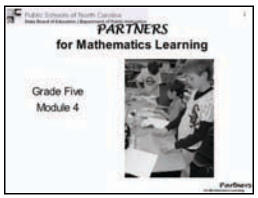
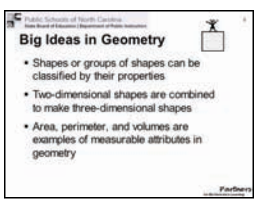
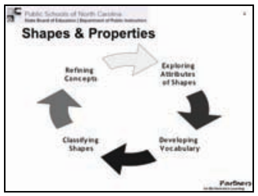
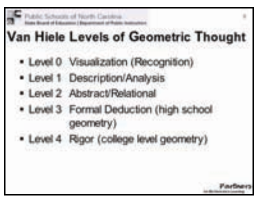
General Materials and Supplies:
Index cards (8x5, 3 per participant) (3x5, one per participant, optional)
Mirrors
Color Square tiles (12 per participant or pair of participants)
Rulers
Cut outs of Assorted Triangles, one set per table
Markers (5 colors)
Cut outs of Assorted Quadrilaterals, one set per table
Waxed paper (≈1 sq. ft. per participant)
Protractor per participant
Scissors
Centimeter Grid Paper (one page per participant)
Example of angle unit (slide 31) cut from index card
Geoboards and bands (one per participant or pair)
Pattern Blocks (at least 2 of each shape per table)
Cm cubes, 36 per pair
Example of activity from slide 34
2 congruent right triangles cut from 9 x 12 paper

Slide	Tasks/Activity	Personal Notes
	(Slide 1) **Partners Grade 5 Module 4** Geometry With the title slide open, ask participants to take a quick look through the geometry sections of the Essential Standards for 4^{th} and 5^{th} grades. Suggest that they look for geometry connections in the measurement strand, as well. Then tell them to think about the process standards as you go through this module and to look for problem solving, reasoning and proof, connections to other strands, representations, and opportunities for communication. Ask them to look particularly for connections to the algebra strand within geometry. Try to spend not much more than 5 minutes looking through the Essential Standards; then tell participants that we will not formally stop to look at these standards during the module as we have a lot to cover, but to keep them in mind as we work through the module.	
	(Slide 2) **Big Ideas in Geometry** Review Big Ideas on this and the next slide quickly. If presenting grades 4 and 5 together, skip to slide 5. Note briefly the difference between attributes and properties - an attribute is a characteristic that can be used to describe an object. Color, shape, size, number of sides, etc. are all attributes. Properties are commonalities that we notice about attributes, i.e., squares all have 4 sides, triangles all have 3 sides, etc. The fact that a rhombus has 4 equal sides is a property that rhombuses have related to the attributes of number of sides and length of sides.	

(Slide 3) **Big Ideas in Geometry**
Be brief in looking at this continuation of the big ideas in geometry.



(Slide 4) **Shapes & Properties**
Review the process children should experience when learning the properties of shapes. Point out that vocabulary must come after exploration so that the terms have meaning. They should not be presented as a list to memorize.

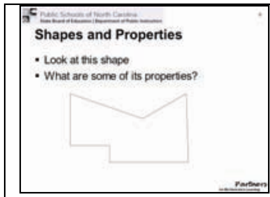


(Slide 5) **Van Hiele Levels of Geometric Thought**
This slide reviews the <u>Van Hiele Levels of Geometric Thought,</u> which were introduced thoroughly last year. Remind participants that the levels are not defined by age, but are developed through experience.

Share these benchmarks for challenging children to reach a new level:

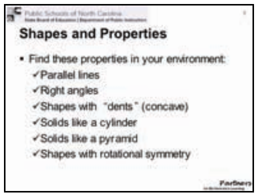
- By third grade, we should begin to challenge children who are ready to reach Level 1 thinking.
- By fifth grade, we may want to attempt to push <u>some</u> students from Level 1 to Level 2, depending on their ability to follow or appreciate logical arguments and their comfort with conjectures and if-then reasoning. Ideas for helping children move from Level 0 to Level 1 and then on to Level 2 when appropriate will be discussed later in this module.

(Slide 6) **Shapes and Properties**
Have participants in table groups list as many properties of this shape as they can. Properties might include: Number of sides (7); Number of angles (7); [Note that the number of sides and the number of angles are the same. Would this be true for all polygons? (yes)]; Some parallel

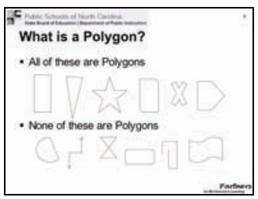


sides; Some perpendicular sides; Some (3) right angles; Two acute angles; One obtuse angle; 2 reflex angles*; No symmetry; Sides are not the same length. Others?

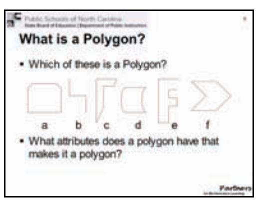
*Some participants may not be familiar with a reflex angle – one that is greater than a straight angle or > 180 °. *Ask Dr. Mat*h at the Math Forum website has some good information about why the angles have the names that they do.



(Slide 7) **Shapes and Properties**
See "Geometry Scavenger Hunt" and "Properties of Shapes Hunt" in handout for other examples of shape and property hunts that students can do. Have participants work in groups to find examples of the properties on the slide or in the handout. They should share some examples that they found. Groups could make a poster of the examples they find. (Don't take time for this now, but make it as a suggestion for their instruction.) How might you alter the search for students at Level 0 or Level 1?

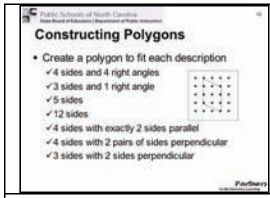


(Slide 8) **What is a Polygon?**
Have participants use these two statements to decide how to define and recognize a polygon, then go to next slide.

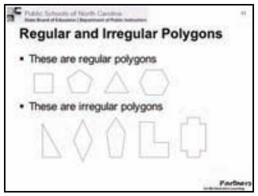


(Slide 9) **What is a Polygon?**
Participants should choose a, c, and f as the polygons. These three figures are closed and have sides that are segments like the figures on the previous slide that were polygons. Figure b is not closed and is only a series of attached line segments. Figure d is closed but has some curved sides. Figure e has no curved sides, but is not closed. Handout, "Defining Polygons," has a similar activity to use with children.

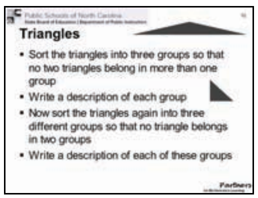
(Slide 10) **Constructing Polygons**
To help children focus on properties, have them do activities like the one on handout, "Constructing Polygons." They are asked to create polygons with particular characteristics on geoboards and to record them on geopaper ("Geoboard Paper" in handout). Does the figure



shown on the slide fit any of the descriptions on the slide? (no) What description might it fit? (e.g., 4 sides with two pairs of sides parallel)

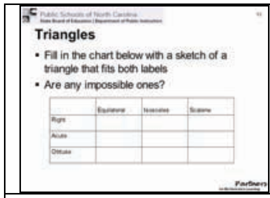


(Slide 11) **Regular and Irregular Polygons**
Have participants use the two descriptions to come up with a definition for a regular polygon. Be sure that their definition contains the ideas that a regular polygon has congruent sides and congruent angles. The irregular polygons may have some congruent sides and/or congruent angles, but do *not* have *all* sides congruent and *all* angles congruent.

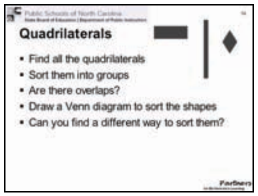


(Slide 12) **Triangles**
This kind of activity is appropriate for instruction at Van Hiele Level 1. Have table groups do this activity using the cut out triangles from handout "Assorted Triangles." Participants, like students, may need a hint to look only at angle size or only at congruent or non-congruent sides, but hold the hint if possible. After descriptions are written, you will have definitions of the six different types of triangles (right, acute, obtuse; equilateral, isosceles, scalene) without having to give definitions to be memorized. The students will have come up with the definitions themselves. You only have to provide the names – the correct vocabulary of triangle identification: A group of triangles with all sides equal are equilateral. Triangles with one right angle are right triangles. Acute triangles have only acute angles, while triangles with one obtuse angle are obtuse triangles. Triangles with at least two congruent sides are isosceles. (This would include the equilateral triangles.) Triangles with no sides congruent are scalene. This vocabulary is not intuitive. It is also important to look at whether or not a triangle could have more than one right angle or obtuse angle. A lot of important information can come from this activity.

(Slide 13) **Triangles**
Have the participants use the information from the previous sorting activity to fill in the chart ("Defining Triangles" in handout) with sketches of triangles. Two are impossible. Which ones? Why? (Equilateral right and equilateral obtuse are impossible. All equilateral triangles have 3 acute angles.)



Note that asking the question "Why are these two impossible?" helps move children from Level 1 to Level 2. Such questions that involve some reasoning will challenge children to move toward Level 2, which focuses on relationships among properties of shapes rather than simple descriptions of properties.

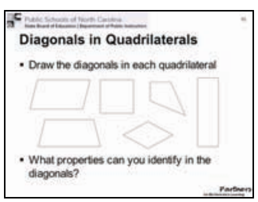


(Slide 14) **Quadrilaterals**
Use the cut out shapes from "Assorted Quadrilaterals" handout. Have participants sort them into groups one way and then at least one more way. They will probably find that the groups overlap, so have them draw a Venn diagram for sorting the shapes. Have them talk about the names for the quadrilaterals.

As they identify the quadrilaterals by name, ask them to focus on the properties of each category of quadrilaterals compared to or related to the properties of another category.

Questions like "what is true for all the rectangles?" and "Let's see if that is true for other rectangles," help move children from Level 0 (identification) to Level 1 (a focus on properties of entire classes of figures).

To help children at Level 1 move toward Level 2, ask questions like "If the sides of a four-sided shape are all congruent, will you always have a square?" (No, you would not because the figure would not necessarily have 4 right triangles.)

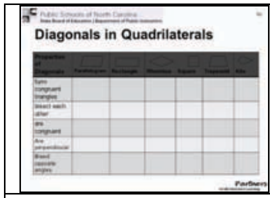


(Slide 15) **Diagonals in Quadrilaterals**
One of the categories of properties of quadrilaterals is their diagonals. Participants may not have a lot of experience thinking about diagonals in quadrilaterals (or other polygons).

Have them predict what properties they might identify in the diagonals. They might say things like diagonals are congruent, diagonals bisect each other, diagonals are perpendicular, etc.

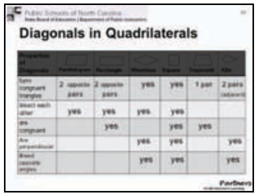
Then show the next slide.

(Slide 16) **Diagonals in Quadrilaterals**
Have participants explore the figures and fill in the chart on handout "Diagonals in



Quadrilaterals, Part 1." Note that the trapezoid is an isosceles one.

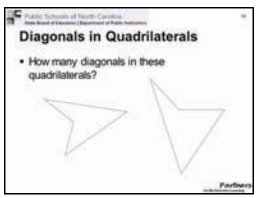
Then show the next slide.



(Slide 17) **Diagonals in Quadrilaterals**
Might the properties of the diagonals in the trapezoid be different if the trapezoid is non-isosceles? Point out that a true exploration would require more than one example of each quadrilateral.

Ask other questions like: "Why do only the rectangle, square, and isosceles trapezoid have congruent diagonals?" and "What do the quadrilaterals that have perpendicular diagonals have in common?"

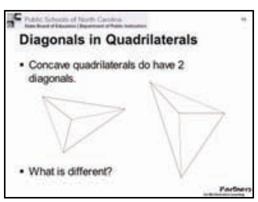
Tell participants that they will do further investigation of quadrilaterals shortly.



(Slide 18) **Diagonals in Quadrilaterals**
There are misconceptions about diagonals in concave polygons (quadrilaterals and others).

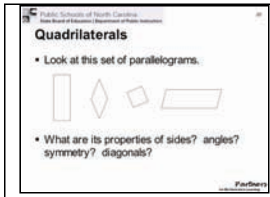
Have the participants draw the diagonals in the quadrilaterals on handout "Diagonals in Quadrilaterals, Part2." Note that one is a kite and another is a non-isosceles trapezoid.

Then discuss the question on the slide before showing the next slide.



(Slide 19) **Diagonals in Quadrilaterals**
Concave quadrilaterals do have 2 diagonals, but one diagonal does not cross the interior of the figure, which has led to the misconceptions. Have participants revisit the definition of a diagonal – a line segment formed by connecting nonadjacent vertices of a polygon (or a line segment connecting vertices of a polygon that do not form a side of the polygon). Given that definition, the diagonal does not have to cross the interior of the figure.

(Slide 20) **Quadrilaterals**
Use handout pages "Properties of Parallelograms," "…Rhombuses," "…Rectangles," and

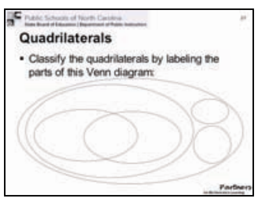


"…Squares" to have participants look at properties of these kinds of quadrilaterals. Perhaps give each table a different quadrilateral to examine. Have available index cards, mirrors, and tracing paper for checking right angles, angle congruence, line and rotational symmetry, and line lengths.

By looking at properties of sides, angles, diagonals, and symmetry, a definition of each kind of quadrilateral can emerge.

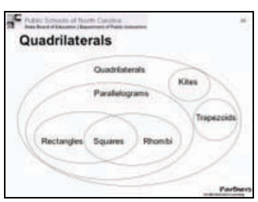
Participants should share their results with the class and come up with a class list on which all can agree. Terminology is important in this activity. Terms including *parallel*, *perpendicular*, *congruent*, *bisect*, and *midpoint* can be clarified during this activity. It is also a good time to introduce symbols such as ≅ for "congruent" and ‖ for "parallel." Similar activities can be done for kites and trapezoids.

See the next slides for a Venn diagram in which the quadrilateral types may be grouped by name, now that definitions for each type have been established.



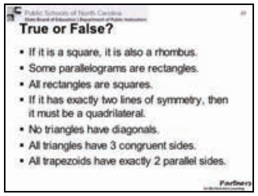
(Slide 21) **Quadrilaterals**
Have participants use handout "The Quadrilateral Family" to show the relationships among the quadrilaterals (rectangle, square, trapezoid, parallelogram, kite, rhombus) by using the Venn diagram.

This diagram includes the convex quadrilaterals. Concave quadrilaterals would not be included in this grouping.

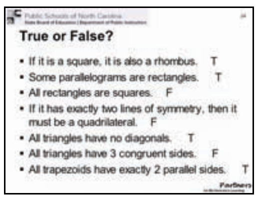


(Slide 22) **Quadrilaterals**
Look at the completed chart.
Have participants talk about the relationships they see.

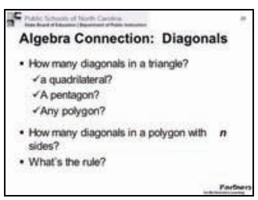
Ask: Why are kites and trapezoids not parallelograms? A kite has two sets of congruent sides but they are not parallel, as they must be in a parallelogram. (Some participants may not be familiar with the kite category of quadrilaterals.) A trapezoid is not a parallelogram because it has only one pair of parallel sides. (Some sources will identify a trapezoid as having "at least one pair of parallel sides" which would put parallelograms in the trapezoid family, but this is not the definition used in the NC Standards.)



(Slide 23) **True or False?**
Have participants respond to each statement as true or false. Have each table choose one statement (with no tables choosing the same statement as another) and justify their decision that the statement is true or false. For example, "Some parallelograms are rectangles" might be proven by showing examples of parallelograms that do not have right angles, but do have the requisite opposite sides parallel. Or "All rectangles are squares" could be disproven similarly by showing rectangles whose sides are not all congruent, but which do have the requisite 4 right angles. See the next slide for true/false solutions.



(Slide 24) **True or False?**
After an activity like this, have participants make their own list of statements, with at least one true statement and one false statement. A list of 4 or 5 statements is a good list. (If you do not have time for them to actually make a new list, be sure to point out that this is a good activity to do with students.)

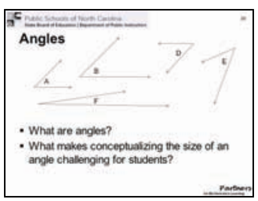


(Slide 25) **Algebra Connection: Diagonals**
Use the activity on handout "Diagonals in Polygons." This activity provides a context for developing algebraic thinking. Children look for patterns in the relationship of the number of sides of a polygon to the number of diagonals that can be drawn in that polygon. Have participants draw the diagonals in the polygons on page 1, fill in the chart on page 2, and try to find the rule for a polygon with *n* sides. The chart should look like this:

Polygon	Type of Polygon	Number of Sides	Number of Diagonals
ABC	triangle	3	0
DEFG	quadrilateral	4	2
HIJKL	pentagon	5	5

MNOPQR	hexagon	6	9
STUVWXY	heptagon	7	14
ABCDEFGH	octagon	8	20
	any polygon	n	[n x (n – 3)]÷ 2 or $\frac{n(n-3)}{2}$

Children will notice the vertical pattern of +2, +3, +4, + 5,…, but this vertical pattern will not help with determining the number of diagonals in any polygon (a polygon with ***n*** sides). It is this functional relationship that we want children to begin to be able to find. Notice that for this activity, the children are asked to write the rule in words before writing it as an equation, and even after writing the equation, they are asked to explain the rule. In this case the rule is half of the product of the number of sides times three less than the number of sides. Each vertex has three less diagonals than the total number of sides (or the total number of vertices) because no diagonal is drawn to the adjacent vertices or to the vertex itself. Since each diagonal touches two vertices but can be counted only once, the product n(n – 3) must be divided by two.



(Slide 26) **Angles**
Ask participants the questions on the slide. Some ideas to offer are below.

Angles are composed of 2 rays joined with a common vertex. Children often have difficulty with angle size because the rays have infinite length. They often mistakenly think the angles with longer sides are bigger. They do not understand that the measure of the angle is the amount of rotation of one ray away from the other. Thus a wide angle with short sides (e.g., angle A) may be considered smaller than a narrow angle with long sides (e.g. angle F). Children also frequently lack exposure to different orientations of the same angle, so might not recognize the similarity in angles A and D or B and E.
Note the connection between the strands of measurement and geometry as participants move through the next activities with angles and angle measurement.

(Slide 27) **Angles**
The measurement strand and the geometry strand intersect at this point. An angle is a *geometric*

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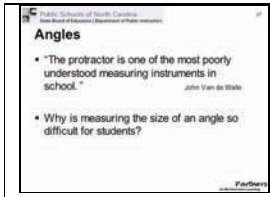
<td>
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<td>attribute and the *protractor* is the tool commonly used to measure that attribute.

Have participants answer the question in groups and report out. They may note the smallness of the degree unit, the dual scale on a protractor, and/or the fact that the degree markings on a protractor do not facilitate visualizing the angle itself.
See the next slide for reasons why the use of a protractor is challenging. Be sure to discuss any that the group did not bring up.</td>
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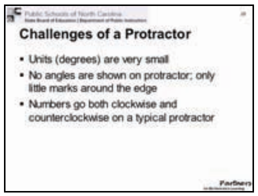
<td>
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<td>(Slide 28) **Challenges of a Protractor**
If these points were mentioned by the groups from the previous slide, do not belabor the points here.

Use this slide to clarify any misunderstandings or to add to the previous conversations. Points may include: The unit size (a degree) is too small to physically use. Note that it would be physically impossible to cut out a single unit of a degree and use it. The 2 sets of numbers on a protractor also make it challenging to use. The children struggle with "Which number do I use?" The next activities will provide opportunities for participants to explore ways to understand angle measure, an angle unit, and how to use a protractor.</td>
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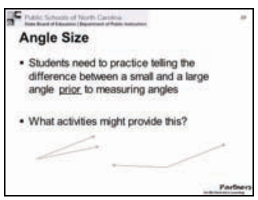
<td>
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<td>(Slide 29) **Angle Size**
Read the top part of the slide, attributing it to John Van de Walle.
Have participants talk in groups to answer the question and then share with the whole group.

Do the following: Use 2 straight edges such as a pair of rulers. Begin with a very small angle and slowly make the opening larger. Ask what this helps children do? (Focus on the size of the angle – how far the straight edge is rotated – not on the length of the sides.)

Using models with different lengths of sides, form angles and compare the size of the angle, not the length of the ray, by overlapping them. Angles may be compared by tracing one and laying it over the other, to compare the spread of the rays of each. Pattern blocks or other shape materials may be used to compare angles by directly placing one angle over another.

See handout, "Comparing Angles," for an activity comparing the angles of polygons by</td>
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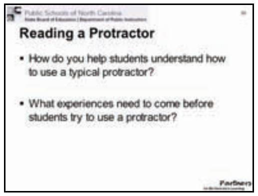
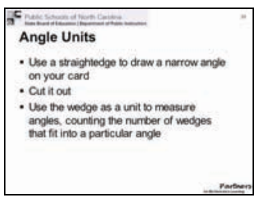
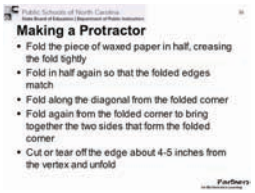
	overlapping the two angles.	
	(Slide 30) **Reading a Protractor** Have participants think about linear or other attributes of objects that children have been using standard tools to measure. How did they come to understand how to use those tools? In many cases, they had experience with non-standard tools before learning about standard ones. They also hopefully had experience with estimating measures before making those measurements. Children need similar experiences when learning to measure angles. For example, children can first "estimate" the size of an angle by identifying it as acute, right or obtuse. Such identification will help children know which scale to use on the protractor. Experience with non-standard measurement of angles will also be helpful in understanding the use of the protractor. The next activities explore non-standard measurement of angles.	
	(Slide 31) **Angle Units** Participants can use an 8x5 index card to create an angle measurement unit following directions on the slide. This activity, from John Van de Walle, provides a non-standard unit that children can create themselves to help them understand the unit of angular measurement. (Have an example to show.) After participants have made their angle unit, have them find a few angles in the room (or use the angles on handout, "Measuring Angles with Your Angle Unit"), estimate the number of angle units in each angle, and then measure the angles by iterating the angle unit and counting the number of units that fit into the angle being measured. Discuss the benefits of this activity. Be sure to include these points: It helps develop the understanding that measuring an angle is measuring the spread of the angle. Angle measurement is like measuring area or length in that you are filling or covering the spread of the angle as you fill or cover an area or length. Select 2 participants who made observably different sizes of angle units. Choose an angle in the room and ask which unit angle would result in the larger number of units, the smaller angle unit or the larger angle unit? (The smaller unit would. Ask participants what principle this demonstrates – Hopefully they will remember the compensatory principle from last year's	

discussions.)



(Slide 32) **Making a Protractor**

Credit this to John Van de Walle. Provide each person with about a square foot of waxed paper. Direct participants to handout, "Making a Protractor." Model each step, being careful to fold so that folded edges match. The completed tool will have 16 unit angles around the center. It makes a protractor with angles units that are 1/8 of a straight angle. Ask participants the benefits of this non–standard tool. It is transparent, and reasonable estimates (½, ⅓, ¼ of a wedge) can be made using it.

Direct participants to use a straight edge to draw a large triangle and then use their waxed paper tool to measure the 3 interior angles in "wedges" and add them for a total number of wedges in a triangle. There should be 8 wedges. Record this as $8^{w.}$ The units are larger and easier to see than on a protractor.

<u>Geometry and Measurement Connection;</u>

Ask why the sum of the angle measures is 8 units for all triangles even though the triangles are different. (8 units form a straight angle, which has 180 degrees)

Next have participants use a straight edge to draw a quadrilateral and measure the interior angles in wedges and add them. There should be 16^{w}.

Tell participants that this activity can be continued for pentagons, hexagons, etc., but do not continue it here. Children would record their data and look for patterns. They should discover the relationship between the number of sides and the sum of the interior angles. [(number of sides – 2) x 8^{w} = sum of interior angles.] When working with a protractor, the unit is degrees. Since there are 180 degrees in a triangle, 8^{w} is replaced by 180 degrees. The superscript ^{w} is replaced by the symbol for degrees (°) since that becomes the unit. The sum of the angles will be explored further later in the module.

(Slide 33) **Making a Tool for Measuring Angles**

Discuss the differences and similarities in the waxed paper tool and a traditional protractor. How would using this non-standard tool help children effectively use and understand the protractor?

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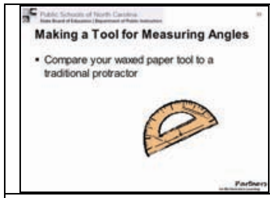
<td>
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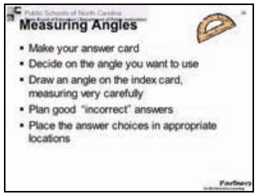
<td>
</td>
<td>(Slide 34) Measuring Angles
Explain that this is an activity for children that provides opportunity to think about angle measurement and provides practice in using a protractor.

Step by step directions are in handout, "Angling for the Correct Measurement."
If you want participants to make one, show your sample and the directions and suggest they make it at a break.</td>
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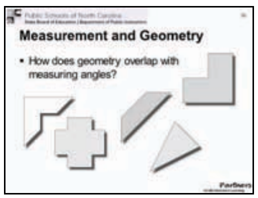
<td>
</td>
<td>(Slide 35) Measurement and Geometry
Have participants respond to the question. They may say things like:
Knowing acute, right and obtuse angles can help children estimate angle size and identify certain geometric figures (e.g., right triangles, etc.).
Knowing there are 180 degrees in a straight angle can help children understand the markings on a protractor.

The process of measuring angles is the same process as measuring length, capacity, or other attributes that can be measured: 1) Select the attribute to measure (in this case, an angle); 2) Choose an appropriate unit to measure that attribute (in this case degrees); and 3) Determine the number of units, using an appropriate tool (in this case, the protractor)</td>
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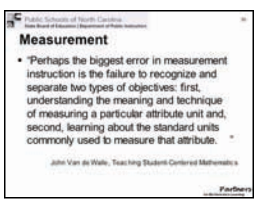
<td>
</td>
<td>(Slide 36) Measurement
Ask: How do the previous activities help children? They begin with focus on what is really being measured: the amount of turn from one side of the angle to the other, rather than the length of the rays. Then the process of measuring angles is explored with non-standard units before using the standard unit of a degree.

Before going to the next slide, show the participants two congruent right triangles cut from 9x12 (or similar) paper. Place a dot on the board or chart paper to provide a vertex. Rip the right</td>
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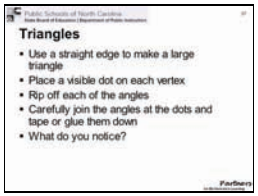
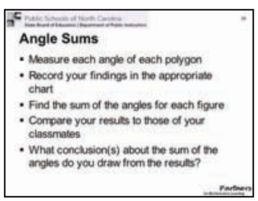
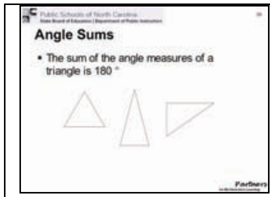
	angle off one of the triangles and place its vertex on the dot. Then rip off another angle and place it next to the first angle with its vertex on the dot. Do the same with the third angle. Participants will notice that the angles have formed a straight angle, which is180°. Draw the rays of the angle which form a straight line, and draw the curved line to show the rotation of one ray from the other. Ask "Do you think that this would happen with all triangles or just with right triangles?" Then show the next slide.	
	(Slide 37) **Triangles** Have participants follow the directions on the slide to explore other kinds of triangles to see if they also form a straight angle if the angles are joined to make a new angle. Step 1-- Encourage a variety of triangles, not just equilaterals. Step 2 -- A visible dot will help to make the vertex apparent when the angles are ripped. Step 3 -- In addition to the dot, it is important to rip the angles rather than cutting them to be able to recognize the original angle of the triangle. Be sure that you have ripped a large enough piece of the angle to manipulate it when taping or gluing down. Step 4 -- Join each of the angles together at the dots and tape or glue them down. Have participants share what happened with the angles of their different triangles. Could a generalization be made about the sum of the angles of a triangle from this experience? This kind of questioning is appropriate for students beginning to operate at Level 2, using some informal deduction.	
	(Slide 38) **Angle Sums** Using handouts (*Triangles, Quadrilaterals, Pentagons, and Hexagons*), have each person at a table choose one triangle, one quadrilateral, one pentagon, and one hexagon – each person choosing different figures from everyone else at the table – and measure the angles of those figures, using a standard protractor. The results should be recorded on the "Angles in Polygons" charts. When all have finished measuring, they should share their results with each other. From those results, they should draw conclusions about the sum of the angles. Ask for conclusions about the sum of the angles of a triangle, which should be at least close to 180°. (You have to allow for small errors in measurement – of up to 5°, but preferably 2° to 3°.)	
	(Slide 39) **Angle Sums** After agreeing on the sum of the angle measures of a triangle, ask for the sum of the angle	

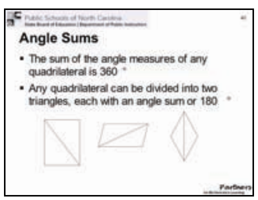


measures of the quadrilaterals. Participants should have found the measures to be 360° or very close to it. (Expect some variation in measures, especially from children. A variation of up to 5°, is acceptable at this point in the children's development, but aim for a variation of 2° or 3° to improve accuracy.)

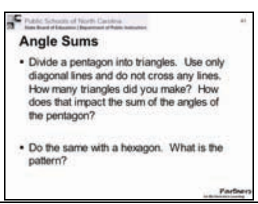
Elicit the sums of the angles of the other polygons. Results should be close to the following:
The sum of the angles of a triangle is 180°.
The sum of the angles of a quadrilateral is 360°.
The sum of the angles of a pentagon is 540°.
The sum of the angles of a hexagon is 720°.

The participants may notice that the sums increase by 180° for each new side. If they do not, ask "What is the relationship between the sum of the angles of the triangle and the sum of the angles of the quadrilaterals?"

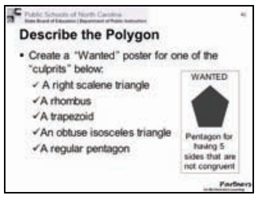
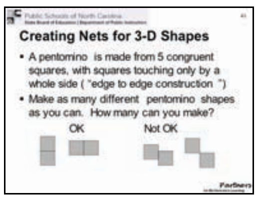
Participants should note that the sum of the angles of the quadrilaterals is twice the sum of the angles of the triangles. Ask why they think this is. Some may note that a quadrilateral can be divided into two triangles. Go to the next slide to illustrate this.

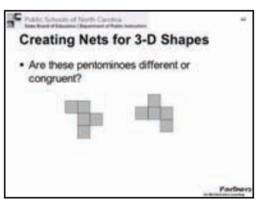


(Slide 40) **Angle Sums**
This slide illustrates that quadrilaterals are in fact two triangles joined together, so that the sum of the angles of the quadrilateral (360°) is the same as the sum of the angles of two triangles (180° + 180°). Ask, "Could something similar be true for pentagons?" Go to the next slide for an exploration of this question.



(Slide 41) **Angle Sums**
Participants will find that a pentagon can be divided into 3 triangles, so the sum of the angles of a pentagon is 3 sets of 180° or 540°. Following this pattern, a hexagon can be divided into 4 triangles or 4 sets of 180° which sums to 720°. Note that not all diagonals are being drawn, since no diagonal lines can be crossed. All diagonals drawn from one vertex would create the appropriate triangles within any given polygon.

<table>
<tr><td></td><td>Activities like this one provide valuable opportunities for students to operate in Level 1, focusing on properties, and to begin to move to Level 2, and develop deductive reasoning. Students could continue the pattern to make conjectures about the sum of the angles of octagons and other polygons.</td><td></td></tr>
<tr><td>
</td><td>(Slide 42) Describe the Polygon
Share this idea with participants. If you have time, you may have them do the activity. They may use construction paper, crayons or markers, and any other useful materials to make an eye-catching poster. The poster should include a picture and a detailed geometric description of the chosen shape, so that classmates can help you "catch" the culprit. Directions for students are on Handout, "Wanted."</td><td></td></tr>
<tr><td>
</td><td>(Slide 43) Creating Nets for 3-D Shapes
Working in table groups, have participants use the color tile squares to find the pentominoes. They should record the ones they find on centimeter grid paper. Do not belabor this. There are 12 pentomino shapes.

Handout pages "Pentomino Shapes" have the pentominoes that can be cut out for use in further activities.</td><td></td></tr>
<tr><td>
</td><td>(Slide 44) Creating Nets for 3-D Shapes
This activity is a good opportunity to revisit transformations and rotations, because a pentomino turned one way is the same pentomino turned another way. Have participants compare these two pentominoes. They are actually the same shape, but the second one has been flipped and then rotated ¼ turn clockwise (or ¾ turn counter-clockwise). Tell them to be sure they have included each pentomino only once. Each group should have found 12 pentominoes.</td><td></td></tr>
<tr><td></td><td>(Slide 45) Creating Nets for 3-D Shapes
Have participants cut out the pentominoes in handout or their representations on the cm grid paper to use to find out which ones fold into a topless box. "Which Pentominoes Make a Topless Box?" in the handouts may be used to record this information. They should identify the bottom of the topless box with an x in that square. Ask participants to look for common features</td><td></td></tr>
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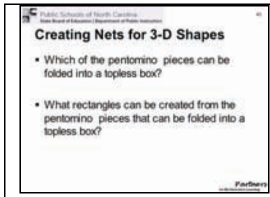
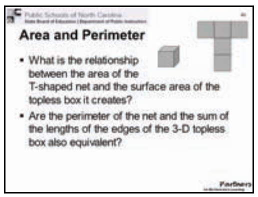
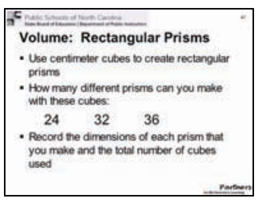


of nets that fold into a topless box.

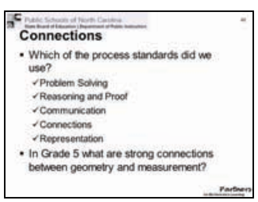
A further activity is handout, "Using Pentominoes," which asks how topless boxes can be cut from a 4x5 sheet of cardboard. The pentomino cut outs can be used to explore this problem. An extension is to have children try to find other size rectangles that could be created from these pentominoes that can make topless boxes. Mention also that children can be challenged to find as many rectangles as they can make using any pentominoes in any combinations. Handout "More Pentomino and Other Net Explorations" lists other activities that explore pentominoes, hexominoes, and other nets.



(Slide 46) **Area and Perimeter**
The value of the area of the net and the surface area of the 3-D topless box are equivalent. Even though surface area is a 3-D characteristic, it is calculated using 2-D area calculations.
The perimeter and the sum of the edge lengths are also equivalent, but the investigation leading to this conclusion is a valuable one.



(Slide 47). **Volume: Rectangular Prisms**
This activity helps lead children to an understanding of what volume means and helps lead them to a formula for finding volume of rectangular prisms.



(Slide 48) **Connections**
Have participants reflect on the activities in this module and share the problem solving that was done. What other process standards were utilized in this module? Participants should see places where they used reasoning and proof (justification); where they communicated ideas, questions, reasoning; when they made connections between strands of math or connections to prior knowledge; and where they used representations to model a situation, to help understand a problem, or to write an equation or other representation in order to solve a problem. Reiterate that the process standards should permeate mathematics instruction on a daily basis, and that they will be evident if effective instruction is going on in the classroom.

	Allow time for participants to respond to both questions at their tables.	
	(Slide 49-52) **Credits and closing slides**	