CONCEPTUAL UNDERSTANDING

- Does correct application of procedures indicate conceptual understanding?

Summarizing the Lesson

Mr. Wicks has been working with his fourth grade students on fractions. He talked about equivalent fractions and and used the example of $2/4$ as another way to name $1/2$. He had students suggest other examples and was pleased that the class seemed to understand.

Regina is one of his best students. She consistently and correctly completes worksheets related to equivalent fractions. She is in the top math group, and Mr. Wicks expects all of the students in that group to score a 100 on the quiz on Friday.

At the end of the class on Thursday, Mr. Wicks asked students to summarize the lesson. "Tell me what you know about equivalent fractions," he said. After a pause, he called on Regina.

Regina's Response

"You just multiply the top number and the bottom number by the same thing," Regina replied as she wrote two examples on her whiteboard.

$2/3 \times 4/4 = 8/12$

$1/2 \times 2/2 = 2/4$

Several other students nodded in agreement.

"Why does this strategy work?" Mr. Wicks asked Regina.

"I don't know but it always works. I can give you some more examples like these." Regina erased her whiteboard and wrote two more examples.

$1/3 \times 3/3 = 3/9$ and $2/5 \times 2/2 = 4/10$

Mr. Wicks' Decision

Mr. Wicks was puzzled when Regina said she didn't know why her strategy worked. Since she was quick to give examples, he thought she understood.

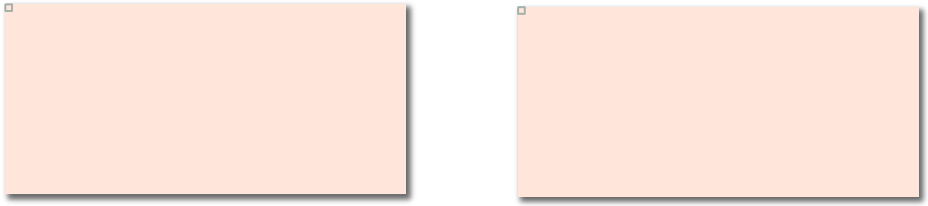
So rather than immediately talking about multiplying by 2/2 or 3/3 or any other representation of 1, he decided to explore further her understanding of equivalence. If one of his top students like Regina did not understand, he suspected that other students also might not have a firm grasp of the concept.

Same or Different?

He drew two identical rectangles on the board and asked,

“If I draw two rectangles that are the same size and shade two-thirds of one of them and shade four-sixths of the other rectangle, will the same amount be shaded?”

“Will they be different fractions?”



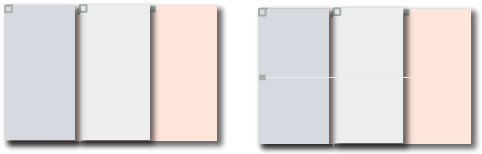
Regina's Response

"The same amount of the rectangles would be shaded." Regina replied. "The numbers are both bigger. Four-sixths is a bigger fraction than two-thirds."

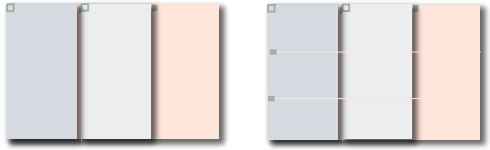
Food for Thought: Mr. Wicks wisely posed a question that had the potential to assess conceptual understanding of equivalence rather than just addressing Regina's lack of understanding of fractions that were equivalent to 1. His discovery that Regina's successful application of procedures (operating with 3/3 or 5/5 or some other fraction that had a value of 1) had led him to a false assumption that she had a solid understanding of equivalence.

Regina's Response

More Food for Thought: Helping Regina connect the model with the notation was one step. Mr. Wicks folded rectangles to illustrate multiplying by 2/2 (each part folded in half) and 3/3 (each part folded into thirds).



2/3 x 2/2 = 4/6



2/3 x 3/3 = 6/9

Addressing this conceptual deficit was critical if Regina and other students were to be successful in subsequent work with fractions.

What Would You Do?

Conceptual Understanding and Procedural Accuracy:

Does this story remind you of any situations in your own classroom?

Do you have students who are able to use procedures but cannot explain “what is happening” (that is, the underlying concepts)?

If you were Regina’s teacher, how would you help her and the others in the class connect the concept of equivalent fractions?

Final Thoughts

Even when students can successfully apply a procedure, it is important that they understand why the procedure works.

Be cautious about assumptions. When we uncover a student's misconceptions or incomplete understandings, it may be wise to check if others also lack solid understanding.

Remember: Conversations with students – before more drill and practice – is a great first step in addressing mistakes.