REASONABLENESS OF RESULTS

How do we help students learn to make judgments about reasonableness of results?

What foundations do students need so that shortcuts do not lead to misdirection?

Continuing the Study of Fractions

Ms. Brownell's class completed the chapter on multiplying fractions last week. On Monday she introduced dividing whole numbers by fractions using the "invert and multiply" strategy. Today students are working in groups to complete a practice page.

As she moved around the classroom, she heard Chris and Nita talking about getting different answers for the same problem:

$$5 \div 2/3$$

Different Results

Chris and Nita each started by writing the problem as

$$5/1 \div 2/3$$

Then Chris wrote $1/5 \times 2/3 = 3/10$

Nita wrote $5/1 \times 3/2 = 15/2$

$$15/2 = 7\ 1/2$$

Shortcut or Reasoning?

“Whoa,” said Chris. “I don’t think your answer is right. I got three-tenths. How did you get seven and a half?”

“It’s your answer that’s wrong,” said Nita. “Think about it. The answer has to be more than five. If you have five cookies and eat two-thirds of each one, there will still be some pieces of the cookies left.”

Chris nodded but looked puzzled. “I did what Ms. Brownell said – when you divide with fractions, you invert and multiply.”

Same Shortcut with Different Results

"We both inverted and multiplied, but you inverted the wrong number, " explained Nita. "Look. When you inverted 5/1 to 1/5 and multiplied, you got 1/5 x 2/3 = 3/10. You are supposed to invert the second number, not the first one. Then when you multiply, you get 5/1 x 3/2 = 15/2 and that means there are seven and a half 2/3s in 5."

Chris grumbled, "That's a dumb rule. It does not make sense that you have to invert the second number rather than the first. Which number you change should not make a difference."

Meanings of Divisions

What examples might a teacher give students to explain the two meanings of division – take away and sharing equally?

How can you model these two meanings?

How would having a story (context) for each model help students make connections?

In what way does a context help students determine if the results of a computation is reasonable?

Reasonableness and Shortcuts

Why are an understanding of these meanings of division and a context for dividing beneficial in helping students recognize how the “invert and multiply” shortcut works?

In what ways could a focus on reasonableness of results help students evaluate if their answers are correct when there is not a context?