ABBY'S RULE

- How do we know when students understand the mathematics that allows rules (algorithms) to work with accuracy?

- Are rules, algorithms, and short cuts the same thing?

Abby Is a Good Problem Solver

Abby, a fifth grade student, is a shy and rarely speaks up unless called upon. But she is a good student and is diligent in completing assignments. She is almost always successful in grasping new mathematics content.

When the class does number talks, Abby is able to think of a second and sometimes a third way of solving the problems.

Occasionally, Abby makes what her teacher considers to be careless mistakes on classwork or homework assignments. When this happens, the teacher admonishes Abby to slow down and check her work because the errors are almost always addition situations.

Abby Is Disappointed

Abby is disappointed when she does not get good grades and is discouraged because she knows all of her number facts.

She does not understand why her answers are sometimes marked wrong when she usually gets all of her work correct.

Here is an example of five problems from Abby's worksheets.

44	19	49	55	27
21	54	23	35	89
53	36	37	27	19
+ 63	+ 18	+ 32	+ 18	+ 26
181	127	132	162	143

In the Grade-level Meeting....

The fifth grade teachers in Abby's school always devote part of each planning meeting to discussions about classroom concerns and potential actions to take. When the topic was addressing students' careless mistakes, her teacher took this sample of Abby's work.

The teacher pointed out that Abby was not consistently "carrying" correctly. Sometimes she wrote down the wrong number and added an incorrect digit to the numbers in the tens or hundreds place. As the group looked at Abby's work, it was clear that this was what was happening.

While Abby's teacher identified the problem, she did not know why Abby made the error in some situations but not in others. She thought these were careless mistakes since more of the fifth grade work focused on multiplication and division.

A Conversation with Abby

The group recommended that she talk with Abby. They discussed "beside the desk conferences" as a strategy in other situations. Rather than just ask about the problems Abby missed, they suggested that the teacher go through all five of the problems and ask Abby to "think aloud."

The teacher asked Abby to explain her answers, adding the columns in each problem out loud. Abby added the numbers in the first two problems and correctly recorded the computations.

In the third problem, Abby orally identified the sum of the numbers in the ones column as 21. Then she wrote down the 2 and carried the 1. The problem had been marked wrong on her paper; and when she got the same answer, Abby looked expectantly at her teacher.

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 49
 23
 37
+ 32
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132
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The Discussion

“In this problem,” Abby’s teacher said pointing to the second computation, “you added the numbers in the ones place and got 27. You wrote down the 7 and carried the 2.”

Abby nodded, and her teacher continued. “In the third problem you added the numbers in the ones place and got 21. That is correct, and you wrote down the 2 and carried the 1 .”

“That’s what you do when you have more than 9 in the ones place,” replied Abby.

“Tell me more about this,” her teacher prompted. She still wanted to understand Abby’s inconsistency.

“When you add, you write down the large number and then carry the small number.” Abby pointed to the problems. “So here you had 27; you write down the 7 and carry the 2. In this one you get 21, so you write down the 2 and carry the 1. That’s the rule.”

```
  19
  54
  36
+ 18
 127
```

```
  49
  23
  37
+ 32
 132
```

Rules Explained

They talked about the final problems; Abby was very consistent in applying *her* rule. The teacher explained that you always record number of ones in the ones place and carry the tens digit. She showed Abby how the pattern works the same way when carrying numbers from the tens place to the hundreds place.

$$\begin{array}{r} 152 \\ +273 \\ \hline 425 \end{array}$$

Abby politely listened, but wondered to herself if this might not always be correct since her teachers in third and fourth grades had not said her rule was wrong.

Often students memorize steps (algorithms) with little underlying understanding or, as Abby demonstrates, they develop their own algorithms that at times result in correct answers and other times fail.

When Telling Doesn't Work

Food for Thought: Algorithms are efficient ways to compute when students understand what each step represents and why they consistently result in accurate results.

Developing that understanding takes more than telling. Students need to engage in activities that allow them to create that understanding through their experiences,

However, teachers must be cautious not to provide opportunities for students to "practice getting it wrong." Abby applied her algorithm consistently in third, fourth, and fifth grades until finally a teacher talked with her about how she arrived at her answers and discovered the inaccurate rule Abby had created for herself. And because she had been applying her rule for so long, Abby was reluctant to give it up.

Steps to Consider

What situations have you experienced in your own classroom that are similar to Abby's story?

How could Abby's teacher have approached the situation differently? That is, when students explain their thinking and their ideas are not correct, how should teachers approach the situation?

Why is it important to be certain the language used to describe steps in an algorithm is accurate and complete. For example, when the sum of numbers in a column is 37, it is not that we "carry the 3" but we record 7 ones and "carry 3 tens."

This story is based on a real classroom experience; Abby's name is changed and the conversation is paraphrased. Source: *INFORMative Assessment: Formative Assessment to Improve Math Assessment, K-6,* Joyner and Muri, Math Solutions, 2011.